

Section 1.2

- 1 - (a) Show that $y^2 + x - 6 = 0$ is an implicit solution to $\frac{dy}{dx} = -\frac{1}{2y}$ on an interval $(-\infty, 6)$

$$y^2 + x - 6 = 0$$

$$y^2 = 6 - x$$

$$y = \pm \sqrt{6 - x}$$

$$y' = -\frac{1}{2\sqrt{6-x}} \quad y_2 = \frac{1}{2\sqrt{6-x}}$$

- (b) Show that

$$xy^3 - x y^3 \sin x = 1$$

$$x y^3 (1 - \sin x) = 1$$

$$y^3 = \frac{1}{x(1 - \sin x)}$$

$$y = \frac{1}{\sqrt[3]{x(1 - \sin x)}} = (x(1 - \sin x))^{-1/3}$$

$$y' = -\frac{1}{3} (x(1 - \sin x))^{-4/3} (1 - (x \cos x + \sin x))$$

$$y' = \frac{x \cos x + \sin x - 1}{3 (x(1 - \sin x))^{4/3}} \quad \text{Slow Down}$$

(2) $y = \sin x + x^7, \quad \frac{d^2 y}{dx^2} + y = x^7 + 42x^5$

$$\frac{dy}{dx} = \cos x + 7x^6$$

$$\frac{d^2 y}{dx^2} = 42x^5 - \sin x$$

Substitute

$$(42x^5 - \sin x) + (\sin x + x^7) = x^7 + 42x^5$$

(3) $\theta = 2e^{2t} - 4e^{3t}, \quad \frac{d^2 \theta}{dt^2} - \theta \frac{d\theta}{dt} + 3\theta = -7e^{3t}$

$$\frac{d\theta}{dt} = 4e^{2t} - 12e^{3t}$$

$$\frac{d^2 \theta}{dt^2} = 8e^{2t} - 36e^{3t}$$

$$(8e^{2t} - 36e^{3t}) - (2e^{2t} - 4e^{3t})(4e^{2t} - 12e^{3t}) + 3(2e^{2t} - 4e^{3t})$$

$$= 8e^{2t} - 36e^{3t} + (4e^{3t} - 2e^{2t})(4e^{2t} - 12e^{3t})$$

$$+ 6e^{2t} - 12e^{3t}$$

$$= 14e^{2t} - 48e^{3t} + (16e^{5t} - 48e^{6t} - 8e^{4t} + 24e^{5t})$$

$$= 14e^{2t} - 48e^{3t} + 40e^{5t} - 48e^{6t} - 8e^{4t}$$

$$= -48e^{6t} + 40e^{5t} - 8e^{4t} - 48e^{3t} + 14e^{2t} \neq -7e^{3t}$$

(4) $e^{2xy} + y^2 = x^2 + 1, \quad \frac{dy}{dx} = \frac{x e^{-2xy} - y}{y e^{-2xy} + x} \quad \text{Implicit}$

$$e^{2xy} (2y + 2x \frac{dy}{dx}) + 2y \frac{dy}{dx} = 2x$$

$$2y e^{2xy} + 2x e^{2xy} \frac{dy}{dx} + 2y \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} (2x e^{2xy} + 2y) = 2x - 2y e^{2xy}$$

$$\frac{dy}{dx} = \frac{x - y e^{2xy}}{x e^{2xy} + y}$$

(5) $4 \sin y + 7xy - x^4 = 3,$

$$y'' = \frac{12x^2 y' + 4(y')^3 \sin y - 14(y')^2}{4x^3 - 7y}$$

$$4 \cos y \frac{dy}{dx} + 7y + 7x \frac{dy}{dx} - 4x^3 = 0$$

$$4 \cos y \frac{dy}{dx} + 7x \frac{dy}{dx} = 4x^3 - 7y$$

$$\frac{dy}{dx} = \frac{4x^3 - 7y}{4 \cos y + 7x}$$

$$\frac{d^2 y}{dx^2} = \frac{(4 \cos y + 7x)(12x^2 - 7 \frac{dy}{dx}) - (4x^3 - 7y)(7 - 4 \sin y \frac{dy}{dx})}{(4 \cos y + 7x)^2}$$

(5) $7 \sin y + 3xy - x^5 = 2$

$$\frac{20x^3 + 7(y')^3 \sin y - 6(y')^2}{5x^4 - 3y} = y''$$

Diff #2

$$7 \cos y \frac{dy}{dx} + 3(y + x \frac{dy}{dx}) - 5x^4 = 0$$

$$7 \cos y \frac{dy}{dx} + 3x \frac{dy}{dx} = 5x^4 - 3$$

$$\frac{dy}{dx} = \frac{5x^4 - 3}{3x - 7 \cos y} \quad \leftarrow \text{For } y'' \text{ Don't Do this}$$

$$7 \cos y y' + 3y + 3xy' - 5x^4 = 0$$

Note! y' is a function on it's own, Deal as if it was it's own variable

Product Rule

$$7y'' \cos y - 7 \sin y (y')^2 + 3y' + 3y'' - 20x^3 = 0$$

$$7y'' \cos y + 3y'' = 7 \sin y (y')^2 + 3y'$$

$$y'' = \frac{7 \sin y (y')^2 + 3y'}{7 \cos y + 3}$$

Not Correct | Ask For Help

(6) Ver

$$\phi(x) = \frac{4}{1 - ce^{5x}}$$

$$\phi(x) = (1 - ce^{5x})^{-1}$$

$$\frac{d\phi}{dx} = (-1)(1 - ce^{5x})^{-2} (0 - (e^{5x})(5))$$

$$\frac{d\phi}{dx} = \frac{20ce^{5x}}{(1 - ce^{5x})^2}$$

(7) $x^m = \phi(x) \quad m x^{m-1} \quad m(m-1)x^{m-2}$

$$5x^2(m^2 x^{m-2} - m x^{m-2}) - 11x(m x^{m-1}) + 3x^m = 0$$

$$5m^2 x^m - 5m x^m - 11m x^m + 3x^m = 0$$

$$x^m (5m^2 - 5m - 11m + 3) = 0$$

$$x^m (5m^2 - 15m - m + 3) = 0$$

$$x^m (5m(m-3) - 1(m-3)) = 0$$

$$x^m (5m-1)(m-3) = 0$$

$$m = \{3, \frac{1}{5}\}$$

$$x^2(m^2 x^{m-2} - m x^{m-2}) + 3x(m x^{m-1}) - 17x^m = 0$$

$$m^2 x^m - m x^m + 3m x^m - 17x^m = 0$$

$$x^m (m^2 + 2m - 17) = 0$$

Section 1.3

Direction Fields

③ $7p - 4p^2 = 0$
 $(7 - 4p)p = 0$

$$p = \left\{ 0, \frac{7}{4} \right\} \quad \frac{7}{4} = 1.75$$

⑥ Find isoclines
set $\frac{dy}{dx} = 0$ solve then
set a variable to infinity

$$\frac{dy}{dx} = 4 - 4y + \frac{2}{x}$$

$$0 = 4 - 4y + \frac{2}{x}$$

$$\lim_{x \rightarrow \infty} 0 = \lim_{x \rightarrow \infty} 4 - 4y + \frac{2}{x} = 4 - 4y$$

$$4 - 4y = 0$$

$$y = 1$$

Section 1.4

①

$$y_{n+1} = y_n + h f(x_n, y_n) ; y(0) = 8$$

$$y_1 = 8 + (0.1)(8(2-8))$$

$$y_1 = 8 - 4.8 = 3.2$$

$$y_2 = (3.2) + (0.1)((3.2)(2-3.2))$$

$$y_2 = 2.816$$

$$y_3 = (2.816) + (0.1)((2.816)(2-2.816))$$

$$y_3 = 2.586$$

$$y_4 = (2.586) + (0.1)((2.586)(2-(2.586)))$$

$$y_4 = 2.4346$$

$$y_5 = (2.4346) + (0.1)((2.4346)(2-(2.4346)))$$

$$y_5 = 2.3288$$

2.3

$$\frac{86r \cdot 3e^t \frac{dy}{dt}}{bx} + \frac{6y \ln t}{y} = \frac{dy}{dt}$$

$$6y \ln t = 3e^t \frac{dy}{dt} - 86$$

Checking equations

Solve for $\frac{dy}{dx}$

$$2t = t e^t \frac{dy}{dt} - 5y \ln t$$

$$4e^t \frac{dy}{dt} = -5y \ln t - 2t$$

$$\frac{dy}{dt} = -\frac{5y \ln t - 2t}{4e^t}$$

See if it fits the $\frac{dy}{dx} + P(x)y = Q(x)$

$$3r = \frac{dr}{d\theta} - 4\theta$$

$$3r + 4\theta^5 = \frac{dr}{d\theta}$$

Q4

$$\frac{dy}{dx} = \frac{y}{x} + 7x + 2$$

$$P(x) = 1/x$$

$$\int 1/x dx = \ln|x|$$

$$-e^{\ln|x|} = \mu(x) = \frac{1}{x}$$

$$\frac{1}{x} \left[\frac{dy}{dx} - \left(\frac{1}{x} \right) \left(\frac{y}{x} \right) \right] = \left(\frac{1}{x} \right) (7x + 2) \left(\frac{1}{x} \right)$$

$$\frac{d}{dx} \left[\frac{y}{x^2} \right] = \frac{1}{x^2} (7x + 2)$$

For $x > 0$

$$\frac{y}{x} = \int 7 + \frac{2}{x} dx$$

For $x < 0$

$$-\frac{y}{x} = -\int 7 + \frac{2}{x} dx$$

$$\frac{y}{x} = 7x + \ln x^2 + C$$

$$y = 7x^2 + x \ln x^2 + xC$$

$$\frac{dy}{dx} = \frac{y}{x} + 2x + 1$$

$$\frac{dy}{dx} - \frac{y}{x} = 2x + 1$$

$$\mu(x) = \left(\frac{1}{x} \right)$$

$$\frac{d}{dx} \left[\frac{y}{x} \right] = \int 2 + \frac{1}{x} dx = 2x$$

$$\frac{y}{x} = 2x + \ln|x| + C$$

$$y = 2x^2 + x \ln|x| + Cx$$

Q5

$$\frac{dr}{d\theta} + r \tan \theta = 8 \sec \theta$$

$$r(\pi) = \tan \theta$$

$$\int \tan \theta d\theta = \ln|\sec \theta|$$

$$\mu = |\sec \theta|$$

$$\frac{d}{d\theta} [r \sec \theta] = \sec \theta (8 \sec \theta)$$

$$r \sec \theta = 8 \int \sec^2 \theta d\theta$$

$$r \sec \theta = 8 \tan \theta + C$$

$$r = \frac{8 \tan \theta + C}{\sec \theta}$$

Q6

$$(t + 4y + 6) dt - dy = 0$$

$$\frac{dy}{dt} = t + 4y + 6$$

$$\frac{dy}{dt} - 4y = t + 6$$

$$P(t) = -4$$

$$\mu = e^{-4t}$$

$$e^{-4t} \frac{dy}{dt} - 4e^{-4t} y = e^{-4t} (t + 6)$$

$$\frac{d}{dt} [y e^{-4t}] = \int e^{-4t} t + 6 e^{-4t} dt$$

$$u = t \quad dv = e^{-4t}$$

$$du = 1 \quad v = -\frac{1}{4} e^{-4t}$$

$$-\frac{1}{4} e^{-4t} t - \int -\frac{1}{4} e^{-4t} dt$$

$$-\frac{1}{4} e^{-4t} t - \frac{1}{16} e^{-4t}$$

$$y e^{-4t} = -\frac{1}{4} e^{-4t} t - \frac{1}{16} e^{-4t} - \frac{3}{2} e^{-4t} + C$$

$$y = -\frac{1}{4} t - \frac{1}{16} - \frac{3}{2} + C e^{4t}$$

$$y = -\frac{1}{4} t - \frac{25}{16} + C e^{4t}$$

Q7

$$\frac{dx}{dt} - \frac{y}{x} = 6 x e^x \quad y(1) = 6e - 3$$

$$P(x) = -\frac{1}{x}$$

$$-\int \frac{1}{x} dx = -\ln|x| = \ln|x|$$

$$e^{\ln|x|} = \frac{1}{x}$$

$$\frac{y}{x} = \int 6 e^x dx$$

$$y = 6 e^x - 3x$$

$$\frac{y}{x} = 6 e^x + C$$

$$y = 6 e^x + Cx$$

Q8

$$\frac{dx}{dx} + 4x - 9e^{-x} = 0 \quad x(0) = 1$$

$$\frac{dx}{dx} = 9e^x$$

$$\frac{d}{dx} [4y e^x] = -9 e^{2x}$$

$$P(x) = 1 \quad 4y e^x = -9 \int e^{2x} dx$$

$$\int dx = e^x \quad 4y e^x = -\frac{9}{2} e^{2x} + C$$

$$y = -\frac{9}{8} e^x + \frac{1}{4} C e^x$$

$$1 = -\frac{9}{8} e^0 + \frac{1}{4} C e^0$$

$$1 + \frac{9}{8} = \frac{1}{4} C$$

$$\frac{17}{8} = \frac{1}{4} C$$

$$\frac{17}{2} = C$$

$$y = \frac{17}{16} e^x - \frac{9}{8} e^x$$

$$\frac{dx}{dx} + 4y - 9e^{-x} = 0 \quad y(0) = 1$$

$$\frac{dx}{dx} + 4y = 9e^{-x}$$

$$P(x) = 4$$

$$e^{\int 4 dx} = e^{4x}$$

$$\frac{d}{dx} [y e^{4x}] = 9 e^{3x}$$

$$y e^{4x} = 9 \int e^{3x} dx$$

$$y e^{4x} = 3 e^{3x} + C$$

$$y = 3 e^{-x} + C e^{-4x} \quad y(0) = 1$$

$$1 = 3 e^0 + C e^0$$

$$1 = 3 + C$$

$$-2 = C$$

$$y = 3 e^{-x} - 2 e^{-4x}$$

$$\frac{dx}{dx} + 4y - 3e^{-2x} = 0 \quad y(0) = -\frac{3}{2}$$

$$\frac{d}{dx} [y e^{4x}] = 3 e^{2x}$$

$$y e^{4x} = \int 3 e^{2x} dx$$

$$y e^{4x} = \frac{3}{2} e^{2x} + C$$

$$y = \frac{3}{2} e^{-2x} + C e^{-4x}$$

$$-\frac{3}{2} = \frac{3}{2} e^0 + C$$

$$-3 = C$$

$$y = \frac{3}{2} e^{-2x} - 3 e^{-4x}$$

Q9

$$t^2 \frac{dx}{dt} + 2tx = t^3 \ln(t) + 6 \quad x(t) = 0$$

$$\frac{dx}{dt} + \frac{2x}{t} = t \ln(t) + \frac{6}{t^2}$$

$$P(x) = \frac{2}{t}$$

$$e^{\int \frac{2}{t} dt} = e^{2 \ln t} = t^2$$

$$\frac{d}{dt} [t^2 x] = t^3 \ln(t) + 6$$

$$t^2 x = \int t^3 \ln(t) + 6 dt$$

$$u = \ln t \quad dv = t^3$$

$$du = \frac{1}{t} \quad v = \frac{1}{4} t^4$$

$$= \frac{1}{4} t^4 \ln(t) - \int \frac{1}{4} t^3 dt = \frac{1}{4} t^4 \ln(t) - \frac{1}{16} t^4 + 6t + C$$

$$x = \frac{1}{4} t^2 \ln(t) - \frac{1}{16} t^2 + \frac{6}{t^2} + \frac{C}{t^2}$$

$$0 = \frac{1}{4} (1)^2 \ln(1) - \frac{1}{16} (1)^2 + \frac{6}{(1)^2} + \frac{C}{(1)^2}$$

$$0 = -\frac{1}{16} + 6 + C$$

$$\frac{1}{16} - 6 = -\frac{95}{16} = C$$

$$t^2 \frac{dx}{dt} + 2tx = t^3 \ln t + 6 \quad x(1) = 0$$

$$\frac{dx}{dt} + \frac{2x}{t} = t^3 \ln t + \frac{6}{t^2}$$

$$P(x) = \frac{2}{t} \quad \int \frac{2}{t} dt = 2 \ln|t|$$

$$\mu(x) = t^2$$

$$t^2 \frac{dx}{dt} + 2tx = t^3 \ln t + 6$$

$$\frac{d}{dt} [t^2 x] = t^3 \ln t + 6$$

$$u = \ln t \quad dv = t^3$$

$$du = \frac{1}{t} \quad v = \frac{1}{4} t^4$$

$$= \frac{1}{4} t^4 \ln(t) - \int \frac{1}{4} t^3 dt = \frac{1}{4} t^4 \ln(t) - \frac{1}{16} t^4 + 6t + C$$

$$x = \frac{1}{4} t^2 \ln t - \frac{1}{16} t^2 + \frac{6}{t^2} + \frac{C}{t^2}$$

$$0 = \frac{1}{4} (1)^2 \ln(1) - \frac{1}{16} (1)^2 + \frac{6}{(1)^2} + \frac{C}{(1)^2}$$

$$0 = -\frac{1}{16} + 6 + C$$

$$\frac{1}{16} - 6 = -\frac{95}{16} = C$$

$$t^2 \frac{dx}{dt} + 4tx = t^3 \ln t + 3 \quad x(1) = 0$$

$$\frac{dx}{dt} + \frac{4x}{t} = t^3 \ln t + \frac{3}{t^2}$$

$$P(x) = \frac{4}{t} \quad \int \frac{4}{t} dt = 4 \ln|t|$$

$$\mu = t^4$$

$$\frac{d}{dt} [x t^4] = t^7 \ln t + 3t$$

$$u = \ln t \quad dv = t^7$$

$$du = \frac{1}{t} \quad v = \frac{1}{8} t^8$$

$$= \frac{1}{8} t^8 \ln t - \frac{1}{8} \int t^7 dt + \frac{3}{2} t^2$$

$$= \frac{1}{8} t^8 \ln t - \frac{1}{64} t^8 + \frac{3}{2} t^2 + C$$

$$x t^4 = \frac{1}{8} t^8 \ln t - \frac{1}{64} t^8 + \frac{3}{2} t^2 + C$$

$$\frac{1}{8} t^4 \ln t - \frac{1}{64} t^4 + \frac{3}{2} t^2 + \frac{C}{t^4} = x(t)$$

$$0 = \frac{1}{8} (1)^4 \ln(1) - \frac{1}{64} (1)^4 + \frac{3}{2} (1)^2 + \frac{C}{(1)^4} = 0$$

$$-\frac{1}{64} + \frac{3}{2} + C = 0$$

$$C = \frac{1}{64} - \frac{3}{2} = -\frac{95}{64}$$

$$t^2 \frac{dx}{dt} + 5tx = t^4 \ln t + 1 \quad x(1) = 0$$

$$\frac{dx}{dt} + \frac{5x}{t} = t^2 \ln t + \frac{1}{t^2}$$

$$P(x) = \frac{5}{t} \quad \int \frac{5}{t} dt = 5 \ln|t|$$

$$\mu = t^5$$

$$\frac{d}{dt} [x t^5] = t^7 \ln t + t^3$$

$$u = \ln t \quad dv = t^7$$

$$du = \frac{1}{t} \quad v = \frac{1}{8} t^8$$

$$= \frac{1}{8} t^8 \ln t - \frac{1}{8} \int t^7 dt + \int t^3 dt$$

$$x t^5 = \frac{1}{8} t^8 \ln t - \frac{1}{64} t^8 + \frac{1}{4} t^4 + C$$

$$x = \frac{1}{8} t^3 \ln t - \frac{1}{64} t^3 + \frac{1}{4} t + \frac{C}{t^5}$$

$$0 = -\frac{1}{64} + \frac{1}{4} + C$$

$$C = \frac{1}{64} - \frac{1}{4} = -\frac{3}{16}$$

$$x(t) = \frac{1}{8} t^3 \ln t - \frac{1}{64} t^3 + \frac{1}{4} t - \frac{3}{16} t^4$$

Q10

$$\frac{dy}{dx} = 5x^3 (5+y^2)^{3/2}$$

$$\int (5+y^2)^{-3/2} dy = \int 5x^3$$

$$\int \frac{1}{(5+y^2)^{3/2}} dy \rightarrow \text{oops} = \frac{x}{5\sqrt{x^2+1}}$$

$$\frac{dy}{dx} = 5x^3 (1+y^2)^{3/2}$$

$$\int \frac{1}{(1+y^2)^{3/2}} dy = \int x^3 dx$$

$$\tan \theta = y$$

$$\frac{dy}{dx} = \sec^2 \theta$$

$$\int \frac{\sec^2 \theta}{(1+\tan^2 \theta)^{3/2}} dy = \int \frac{\sec^2 \theta}{\sec^3 \theta} d\theta = \int \frac{1}{\sec \theta}$$

$$\int \cos \theta d\theta = \sin(u) = \frac{x}{\sqrt{x^2+1}}$$

$$\sin(\tan^{-1}(y)) = \frac{y}{\sqrt{y^2+1}}$$

$$\tan^{-1}(y) = \sin^{-1}(\frac{y}{\sqrt{y^2+1}})$$

$$y = \tan(\sin^{-1}(\frac{y}{\sqrt{y^2+1}}))$$

Q11

$$\int (y+1) dy = \int \frac{5x^2-x-3}{x^2(x+1)} dx$$

$$\int -\frac{3}{x} + \frac{3}{x+1} + \frac{2}{x} dx = \frac{3}{2} + 3 \ln|x+1| + 2 \ln|x|$$

$$= \frac{1}{2} y^2 + 2$$

$$3 \ln|x+1| + \ln|x| + \frac{2}{x} = \frac{1}{2} y^2 + y + C$$

$$3 \ln(2) + \ln(1) + \frac{2}{1} = \frac{1}{2} (2)^2 + 2 + C$$

$$3 \ln(2) + \frac{2}{1} = \frac{1}{2} (2)^2 + 2 + C$$

$$3 \ln(2) - 2 = C$$

Q12

$$\frac{1}{\theta} \frac{dy}{d\theta} = \frac{y \sin \theta}{y^{2/4}}$$

$$\left(\frac{y^{1/4} + y}{y} \right) dy = (\theta \sin \theta) d\theta$$

$$\int y + \frac{1}{y} dy = \int \theta \sin \theta d\theta$$

$$u = \theta \quad dv = \sin \theta$$

$$du = 1 \quad v = -\cos \theta$$

$$-\theta \cos \theta + \int \cos \theta d\theta = -\theta \cos \theta + \sin \theta$$

$$y(\pi) = 1$$

$$-\theta \cos \theta + \sin \theta = \frac{1}{2} y^2 + \ln|y| + C$$

$$-\pi \cos \pi + \sin \pi = \frac{1}{2} (1)^2 + \ln|1| + C$$

$$\pi = \frac{1}{2} + C$$

$$\pi - \frac{1}{2} = C$$

$$(xy^2 + 2y^2)dy - 5y dx = 0$$

$$xy^2 dy + 2y^2 dy = 5y dx$$

$$xy dy + 2y dy = dx$$

$$xy + 2y = \frac{dx}{dy}$$

$$(x+2)y = \frac{dx}{dy}$$

$$\left(\frac{1}{x+2}\right) dx = \left(\frac{1}{y}\right) dy$$

2.4

Q2

Determine if it's exact

$$\frac{\partial}{\partial y} (4x^3y + 9) dx + (x^4 - 7) dy = 0$$

$$\frac{\partial M}{\partial y} = 4x^3$$

$$\frac{\partial N}{\partial x} = 4x^3$$

$$(2xy + 9) dx + (x^2 - 4) dy = 0$$

$$2x \checkmark = 2x$$

$$\int 2xy + 9 dx = x^2y + 9x + C(y)$$

$$\frac{\partial}{\partial y} [x^2y + 9x + C(y)] = x^2 + C_y(y)$$

$$x^2 - 4 = x^2 + C_y(y)$$

$$-4 = C_y(y)$$

$$-4y = C(y)$$

$$F(x, y) = x^2y + 9x - 4y = C$$

$$\int 2xy + 8 dx$$

$$x^2y + 8x + C(y)$$

$$x^2 + C_y = x^2 - 4$$

$$-4y = C$$

$$x^2y + 8x - 4y = C$$

Q3

$$(\cos x \sin y + 6x) dx + (\sin x \cos y + 3y) dy = 0$$

$$\frac{\partial M}{\partial y} = \cos(x) \cos(y)$$

$$\frac{\partial N}{\partial x} = \cos(x) \cos(y)$$

$$\int \cos(x) \sin(y) + 6x dx$$

$$\sin(x) \sin(y) + 3x^2 + C(y)$$

$$\frac{\partial}{\partial y} [\sin(x) \sin(y) + 3x^2 + C(y)]$$

$$\sin(x) \cos(y) + C_y = \sin(x) \cos(y) + 2y$$

$$C_y = 2y$$

$$C = y^2$$

$$\sin(x) \sin(y) + 3x^2 + y^2 = C$$

Q4

$$\left(\frac{1}{x} + 6y^2x\right) dx + (6yx^2 + \sin y) dy = 0$$

$$\frac{\partial M}{\partial y} = 12yx$$

$$\frac{\partial N}{\partial x} = 12yx$$

$$\int \frac{1}{x} + 6y^2x dx$$

$$\ln|x| + 3y^2x^2 + C(y)$$

$$\frac{\partial}{\partial y} [\ln|x| + 3y^2x^2 + C(y)]$$

$$6yx^2 + C_y = 6yx^2 + \sin y$$

$$C_y = \sin y$$

$$y = \cos y$$

$$\ln|x| + 3y^2x^2 + \cos y = C$$

$$y(1) = \pi$$

$$\ln(1) + 3(\pi)^2(1)^2 + \cos(\pi) = C$$

$$0 + 3\pi^2 - 1 = C$$

Q4

$$\left(\frac{1}{x} + 6y^2x\right) dx + (6yx^2 - \sec^2 y) dy$$

$$y(1) = \pi$$

$$\int \frac{1}{x} + 6y^2x dx =$$

$$= \ln|x| + 3y^2x^2 + C_y$$

$$\frac{\partial}{\partial y} [\ln|x| + 3y^2x^2 + C_y]$$

$$= 6yx^2 + C_y$$

$$6yx^2 + C_y = 6yx^2 - \sec^2 y$$

$$C_y = -\sec^2 y$$

$$y = -\tan y$$

$$\ln x + 3y^2x^2 - \tan y = C$$

$$\ln(1) + 3(\pi)^2(1)^2 - \tan(\pi) = C$$

$$3\pi^2 = C$$

$$\ln x + 3y^2x^2 - \tan y = 3\pi^2$$

Q5

$$\left(7ye^{7xy} - \frac{9}{y}\right) dx + \left(7xe^{7xy} + \frac{9x}{y^2}\right) dy$$

$$\int 7ye^{7xy} - \frac{9}{y} dx$$

$$e^{7xy} - \frac{9x}{y} + C(y)$$

$$\frac{\partial}{\partial y} \left[e^{7xy} - \frac{9x}{y} + C(y) \right]$$

$$7xe^{7xy} + \frac{9x}{y^2} C_y = 7xe^{7xy} + \frac{9x}{y^2}$$

$$C_y = 0$$

$$\int 7ye^{7xy} - \frac{9}{y} dx$$

$$7y \int e^{7xy} - \frac{9}{y} dx$$

$$u = 7xy \quad \cdot = e^{7xy} - \frac{9x}{y^2} + C(y)$$

$$\frac{du}{dx} = 7y = \frac{\partial}{\partial y} \left[e^{7xy} - \frac{9x}{y} + C(y) \right]$$

$$\frac{1}{7y} du = dx \quad 7xe^{7xy} + \frac{9x}{y^2} + C_y =$$

$$7xe^{7xy} + \frac{9x}{y^2} + C_y = 7xe^{7xy} + \frac{9x}{y^2}$$

$$C_y = 0$$

$$\left(8ye^{8xy} - \frac{9}{y}\right) dx + \left(8xe^{8xy} + \frac{9x}{y^2}\right) dy$$

$$\int 8xe^{8xy} + \frac{9x}{y^2} dy$$

$$\frac{\partial}{\partial x} \left[e^{8xy} - \frac{9x}{y} + C(x) \right]$$

$$8ye^{8xy} - \frac{9}{y} + C_x = 8ye^{8xy} + \frac{9}{y}$$

$$C_x = 0$$

$$C(x) = 0$$

$$e^{8xy} - \frac{9x}{y} =$$

$$e^{8(1)(3)} - \frac{9(3)}{1} = e^{24} - 27$$

Not the same C's

3. 2

$$h = .1$$

$$x = 4$$

$$\frac{4}{n} =$$

$$7\left(1 - \frac{h}{2}\right)^{4/h}$$

$$y_n - \frac{1}{2} y_n h$$

$$y_{n+1} = e^x + \frac{h}{2} [e^x + e^{x+h}]$$

$$y_{n+1} = e^x + \frac{h}{2} e^x + \frac{h}{2} e^{x+h}$$

$$y_{n+1} = e^x (h/2 + e^h)$$

$$y_{n+1} = y_n + \frac{h}{2} [y_n + y_{n+1}]$$

$$y_{n+1} = y_n + \frac{h y_n}{2} + \frac{h y_{n+1}}{2}$$

$$y_{n+1} - \frac{h y_{n+1}}{2} = y_n + \frac{h y_n}{2}$$

$$y_{n+1} \left(1 - \frac{h}{2}\right) = y_n \left(1 + \frac{h}{2}\right)$$

$$y_{n+1} = y_n \left(\frac{1 + \frac{h}{2}}{1 - \frac{h}{2}}\right)$$

3.2

in 8 L/min at 0.05 kg/L

$$V(t) = 100 + 0t$$

$$x(t) = ?$$

$$\frac{dx}{dt} = 0.4 - 0.5 \left(\frac{x}{100} \right)$$

$$\text{in} - \text{out} = \frac{dx}{dt}$$

$$\text{in} = (\text{Flow in})(\text{dilution})$$

$$\text{out} = (\text{Flow out}) \frac{x}{\text{Tank Vol} + (\text{in} - \text{out})t}$$

$$\frac{dx}{dt} = 0.4 - 0.08x$$

$$\frac{dx}{dt} + 0.08x = 0.4$$

$$\rho(x) = 0.08 \quad \mu(t) = e^{0.08t}$$

$$\frac{d}{dt} [x e^{0.08t}] = 0.4 e^{0.08t}$$

$$x e^{0.08t} = 5 e^{0.08t} + C$$

$$x(t) = 5 + C e^{-0.08t}$$

$$x(0) = 50$$

$$0.5 = 5 + C e^0$$

$$x(t) = 5 - 4.5 e^{-0.08t}$$

$$0.03 = 5 - 4.5 e^{-0.08t}$$

→ Total salt in tank

$$Q(x) = 0.05 - 0.045 e^{-0.08t}$$

→ concentration of salt in the tank

$$0.03 = 0.05 - 0.045 e^{-0.08t}$$

$$\frac{4}{9} = e^{-0.08t}$$

$$\ln(4/9) = -0.08t$$

in 3 L/min

Tank 100 L

out 2 L/min

$$\frac{dx}{dt} = 3 \cdot 0.6 - \frac{x}{100+t} \cdot 2$$

$$\frac{dx}{dt} = 1.8 - \frac{2x}{100+t}$$

$$\frac{dx}{dt} + \frac{2x}{100+t} = 1.8$$

$$P(t) = \frac{2}{100+t} \quad 2 \int \frac{1}{100+t} = 2 \ln|100+t|$$

vol > 0

$$\mu(t) = (100+t)^2$$

$$[(100+t)^2 x] \frac{d}{dx} = 1.8 (100+t)^2$$

$$(100+t)^2 x = 0.6 (100+t)^3 + C$$

$$x(t) = 0.6 (100+t) + \frac{C}{(100+t)^2}$$

$$0 = 0.6 (100) + \frac{C}{10000}$$

$$-60 = C/10000$$

$$-600,000 = C$$

$$x(t) = 0.6 (100+t) - \frac{600,000}{(100+t)^2}$$

$$\frac{x(t)}{(100+t)} = 0.6 - \frac{600,000}{(100+t)^3}$$

$$Q(t) = 0.6 - \frac{600,000}{(100+t)^3}$$

$$0.5 = 0.6 - \frac{600,000}{(100+t)^3}$$

$$0.1 = \frac{600,000}{(100+t)^3}$$

$$0.1 (100+t)^3 = 600,000$$

$$100+t = 181.71 \dots$$

$$t = 81.71$$

in → 6 L/min 0.4 kg/L

Tank 100 L

out 5 L/min

$$\frac{dx}{dt} = 6 \cdot 0.4 - \frac{5x}{100+t}$$

$$\frac{dx}{dt} = 2.4 - \frac{5x}{100+t}$$

$$\left[x (100+t)^5 \right] = 2.4 \int (100+t)^5 dt$$

$$0.4 (100+t)^6 + C = x (100+t)^5$$

$$x(t) = 0.4 (100+t) + \frac{C}{(100+t)^5}$$

$$0 = 0.4 (100) + \frac{C}{100^5}$$

$$0.2 = 0.4 - \frac{100^5 \times 40}{(100+t)^6}$$

$$\frac{0.2}{100^5 \times 40} = \frac{1}{(100+t)^6}$$

$$\sqrt[6]{\frac{100^5 \times 40}{0.2}} = 100+t$$

3.

4.2

① $21y'' - 11y' - 2y = 0$

- Auxiliary equation

$$21r^2 - 11r - 2 = 0$$

$$\begin{array}{c} -42 \\ \wedge \\ 2 \quad 21 \\ \wedge \\ 7 \quad 3 \end{array}$$

$$21r^2 - 14r + 3r - 2 = 0$$

$$7r(3r - 2) + 1(3r - 2) = 0$$

$$(3r - 2)(7r + 1)$$

$$r = \{2/3, -1/7\}$$

$$y(t) = C_1 e^{2/3 t} + C_2 e^{-1/7 t}$$

② $3z'' + 3z' - z = 0$

$$t = \frac{-3 \pm \sqrt{9+12}}{6} = \frac{-3 \pm \sqrt{21}}{6}$$

$$y(t) = C_1 e^{\frac{-3-\sqrt{21}}{6} t} + C_2 e^{\frac{-3+\sqrt{21}}{6} t}$$

③ $25w'' + 60w' + 36w = 0$

$$25a^2 + 60a + 36 = 0$$

$$\begin{array}{c} 900 \\ \wedge \\ 30 \cdot 30 \end{array}$$

$$25a^2 + 30a + 30a + 36 = 0$$

$$5a(5a + 6) + 6(5a + 6) = 0$$

$$(5a + 6)(5a + 6) = (5a + 6)^2$$

$$w(t) = C_1 e^{-6/5 t} + C_2 t e^{-6/5 t}$$

④ $y'' + 4y' = 0 \quad y(0) = 11, \quad y'(0) = -28$

$$r^2 + 4r = 0$$

$$r(r + 4) = 0$$

$$r = \{0, -4\}$$

$$y(t) = C_1 e^{-4t} + C_2$$

$$y'(t) = -4C_1 e^{-4t}$$

$$-28 = -4 e^{-4(0)} C_1$$

$$-28 = -4 C_1$$

$$7 = C_1$$

$$y(t) = 7 e^{-4t} + C_2$$

$$11 = 7 e^{-4(0)} + C_2$$

$$11 = 7 + C_2$$

$$4 = C_2$$

$$y(t) = 7 e^{-4t} + 4$$

⑤ $y'' + y' = 0 \quad y(0) = 1, \quad y'(0) = 6$

$$a^2 + a = 0$$

$$a = \{0, -1\}$$

$$y(t) = C_1 e^{-t} + C_2$$

$$y'(t) = -C_1 e^{-t}$$

$$6 = -C_1$$

$$y(t) = -6 e^{-t} + C_2$$

$$1 = -6 + C_2$$

$$7 = C_2$$

$$y(t) = -6 e^{-t} + 7$$

⑥ $y'' - 4y' + 3y = 0 \quad y(0) = -\frac{4}{5}, \quad y'(0) = -\frac{18}{5}$

$$a^2 - 4a + 3 = 0$$

$$(a - 1)(a - 3) = 0$$

$$a = \{1, 3\}$$

$$y(t) = C_1 e^t + C_2 e^{3t}$$

$$y'(t) = C_1 e^t + 3C_2 e^{3t}$$

$$\begin{array}{l} -\frac{18}{5} = C_1 + 3C_2 \\ -\frac{4}{5} = C_1 + C_2 \end{array} \rightarrow \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -\frac{18}{5} \\ -\frac{4}{5} \end{bmatrix}$$

$$y(t) = \frac{3}{5} e^t - \frac{7}{5} e^{3t}$$

⑦ Determine if functions y_1 and y_2 are linearly independent on the interval $(0, 1)$

$$y_1 = \sin t \cos t$$

$$y_2 = 6 \sin 2t$$

$$\frac{6 \sin(2t)}{\sin t \cos t} = \frac{6(2 \cos t \sin t)}{\cos t \sin t} = 12$$

⑧ $y_1 = e^{7t} \quad y_2 = e^{5t}$

⑨ $\tan^2 - \sec^2 t$

4.3

① $y'' - 4y' + 20y = 0$

Aux

$$r = \frac{4 \pm \sqrt{16 - 4 \cdot 20}}{2} = \frac{4 \pm i8}{2} = 2 \pm 4i$$

$$y(t) = C_1 e^{2t} \cos 4t + C_2 e^{2t} \sin 4t$$

② $4y'' - 4y' + 2y = 0$

$$r = \frac{4 \pm \sqrt{16 - 4 \cdot 4 \cdot 2}}{2 \cdot 4} = \frac{4 \pm 4i}{8} = \frac{1}{2} \pm \frac{1}{2}i$$

$$y(t) = C_1 e^{\frac{1}{2}t} \cos \frac{1}{2}t + C_2 e^{\frac{1}{2}t} \sin \frac{1}{2}t$$

③ $y'' + 8y' + 25y = 0 \quad y(0) = 2, \quad y'(0) = -6$

$$r = \frac{-8 \pm \sqrt{64 - 4(25)}}{2} = \frac{-8 \pm 6i}{2} = -4 \pm 3i$$

$$y(t) = C_1 e^{-4t} \cos(3t) + C_2 e^{-4t} \sin(3t)$$

$$\begin{aligned} y'(t) &= -4C_1 e^{-4t} \cos(3t) - 3C_1 e^{-4t} \sin(3t) \\ &\quad - 4C_2 e^{-4t} \sin(3t) + 3C_2 e^{-4t} \cos(3t) \\ &= e^{-4t} (-4C_1 \cos(3t) + 3C_2 \cos(3t) \\ &\quad - 3C_1 \sin(3t) - 4C_2 \sin(3t)) \end{aligned}$$

$$= e^{-4t} (\cos(3t) (-4C_1 + 3C_2) + \sin(3t) (-3C_1 - 4C_2))$$

$$y'(0) = -6$$

$$e^{-4(0)} (C_1 \cos(3(0)) (-4C_1 + 3C_2) + \sin(3(0)) (-3C_1 - 4C_2))$$

$$y'(0) = -4C_1 + 3C_2 = -6$$

$$y(0) = C_1 + 0 = 2$$

$$y(t) = 2 e^{-4t} (\cos(3t)) + \frac{2}{3} e^{-4t} \sin(3t)$$

④ $y'' + 36y = 0 \quad y(0) = 5 \quad y'(0) = 1$

aux

$$r^2 + 36 = 0$$

$$r = \pm 6i$$

$$y(t) = C_1 \cos 6t + C_2 \sin 6t$$

$$y'(t) = -C_1 6 \sin 6t + C_2 6 \cos 6t$$

$$y(0) = 5$$

$$5 = C_1 + 0$$

$$1 = 0 + C_2 6 \cos 6t$$

$$y(t) = 5 \cos 6t + \frac{1}{6} \sin 6t$$

4. 4

(2.4)

$$y'' - y' + 36y = 6 \sin 6t$$

$$y(t) = A \sin(6t) + B \cos(6t)$$

$$y'(t) = 6A \cos(6t) - 6B \sin(6t)$$

$$y''(t) = -36A \sin(6t) - 36B \cos(6t)$$

$$-36A \sin(6t) - 36B \cos(6t) - 6A \cos(6t) + 6B \sin(6t)$$

$$-6A \cos 6t + 6B \sin 6t = 6 \sin(6t)$$

(2.6)

$$y'' - 7y' + 2y = xe^x$$

$$y(x) = (Ax + B)e^x$$

$$y'(x) = e^x(Ax + B) + e^x(A)$$

$$y'(x) = e^x(Ax + B + A)$$

$$y''(x) = e^x(Ax + B + A) + e^x(A)$$

$$y''(x) = e^x(Ax + B + 2A)$$

$$e^x(Ax + B + 2A - 7Ax - 7B - 7A + 2Ax + 2B)$$

$$= e^x(-4Ax - 4B - 3A) = xe^x$$

$$-4Ax - 4B - 3A = x$$

$$-\frac{1}{4} = A$$

$$-4B - 3A = 0$$

$$-4B - (-\frac{1}{4}) = 0$$

$$-4B + \frac{1}{4} = 0$$

$$4B = \frac{1}{4}$$

$$B = \frac{1}{16}$$

$$y'' - 8y' + 9y = xe^x$$

$$y(x) = (Ax + B)e^x$$

$$y' = e^x(Ax + B + A)$$

$$y'' = e^x(Ax + B + 2A)$$

$$e^x(Ax + B + 2A - 8Ax - 8B - 8A + 9Ax + 9B)$$

$$e^x(Ax - 8Ax + 9Ax + B - 8B + 9B + 2A - 8A)$$

$$e^x(2Ax + 2B - 6A) = xe^x$$

$$A = \frac{1}{2}$$

$$2B - 6A = 0$$

$$2B - 6 \cdot \frac{1}{2} = 0$$

$$2B - 3 = 0$$

$$2B = 3$$

$$B = \frac{3}{2}$$

$$y(x) = (\frac{1}{2}x + \frac{3}{2})e^x$$

(2.7)

$$y'' - 4y' + 6y = xe^x$$

$$e^x(Ax + B + 2A - 4Ax - 4B - 4A + 6Ax + 6B)$$

$$= e^x(3Ax + 3B - 2A) = xe^x$$

$$A = \frac{1}{3}$$

$$3B - 2(\frac{1}{3}) = 0$$

$$3B - \frac{2}{3} = 0$$

$$3B = \frac{2}{3}$$

$$B = \frac{2}{9}$$

$$y(x) = (\frac{1}{3}x + \frac{2}{9})e^x$$

(2.8)

$$x''(t) - 14x'(t) + 44x(t) = 5te^{7t}$$

$$x(t) = (At^3 + Bt^2)e^{7t}$$

$$x'(t) = e^{7t}(7At^3 + 7Bt^2 + 3At^2 + 2Bt)$$

$$x'(t) = e^{7t}(7At^3 + 3At^2 + 7Bt^2 + 2Bt)$$

$$x''(t) = e^{7t}(49At^3 + 21At^2 + 49Bt^2 + 14Bt + 21At^2 + 6At + 14Bt + 2B)$$

$$x''(t) = e^{7t}(49At^3 + 42At^2 + 6At + 49Bt^2 + 28Bt + 2B)$$

$$(49At^3 + 42At^2 + 6At + 49Bt^2 + 28Bt + 2B - 98At^3 - 42At^2 - 98Bt^2 - 28Bt + 49At^3 + 49Bt^2) = 5te^{7t}$$

$$5te^{7t} = (6At + 2B)e^{7t}$$

$$B = 0$$

$$A = \frac{5}{6}$$

$$(\frac{5}{6}t^3)e^{7t}$$

(2.9)

$$x''(t) - 12x'(t) + 36x(t) = 5te^{6t}$$

$$x(t) = (At^3 + Bt^2)e^{6t}$$

$$x'(t) = e^{6t}(6At^3 + 6Bt^2 + 3At^2 + 2Bt)$$

$$x'(t) = e^{6t}(6At^3 + 3At^2 + 6Bt^2 + 2Bt)$$

$$x''(t) = e^{6t}(36At^3 + 18At^2 + 36Bt^2 + 12Bt + 18At^2 + 6At + 12Bt + 2B)$$

$$x''(t) = e^{6t}(36At^3 + 36At^2 + 6At + 36Bt^2 + 24Bt + 2B)$$

$$e^{6t}(36At^3 + 36At^2 + 6At + 36Bt^2 + 24Bt + 2B - 72At^3 - 36At^2 - 72Bt^2 - 24Bt + 36At^3 + 36Bt^2)$$

$$(6At + 2B)e^{6t} = 5te^{6t}$$

$$A = \frac{5}{6} \quad B = 0$$

$$x(t) = \frac{5}{6}t^3e^{6t}$$

(2.10)

$$y'' + 4y' + 8y = 325e^{2t} \cos 5t$$

$$y(t) = e^{2t}(A \cos 5t + B \sin 5t)$$

$$y'(t) = e^{2t}(2A \cos 5t + 2B \sin 5t - 5A \sin 5t + 5B \cos 5t)$$

$$= e^{2t}(2A \cos 5t - 5A \sin 5t + 2B \sin 5t + 5B \cos 5t)$$

$$y''(t) = e^{2t}(4A \cos 5t - 10A \sin 5t + 4B \sin 5t + 10B \cos 5t - 10A \sin 5t - 25A \cos 5t + 10B \cos 5t - 25B \sin 5t)$$

$$= e^{2t}(-21A \cos 5t - 20A \sin 5t + 20B \cos 5t - 21B \sin 5t)$$

$$e^{2t}(-21A \cos 5t - 20A \sin 5t + 20B \cos 5t - 21B \sin 5t + 8A \cos 5t - 20A \sin 5t + 20B \cos 5t + 8B \sin 5t)$$

$$e^{2t}(-5A \cos 5t - 40A \sin 5t + 40B \cos 5t - 5B \sin 5t)$$

$$((-5A + 40B) \cos 5t + (-40A - 5B) \sin 5t) e^{2t}$$

$$\begin{cases} -5A + 40B = 325 \\ -40A - 5B = 0 \end{cases} = \begin{matrix} A = -1 \\ B = 8 \end{matrix}$$

$$y(t) = e^{2t}(8 \sin 5t - \cos 5t)$$

(12)

$$y'' + 10y' + 41y = 2e^3 e^{-5t} \sin 4t$$

$$r^2 + 10r + 41 = 0$$

$$b^2 - 4ac$$

$$100 - 4(1)(41) = -64$$

$$r = \frac{-10 \pm \sqrt{-64}}{2} = \frac{-10 \pm 8i}{2} = -5 \pm 4i$$

4.5

Q2

$$y'' - y = 2e$$

$$y_p = Ae + B$$

$$y' = A$$

$$y'' = 0$$

$$0 - Ae + B = 2e$$

$$B = 0 \quad A = -2$$

$$y_p = -2e$$

$$y_o = C_1 e^t + C_2 e^{-t} - 2e$$

Q3

$$y'' - y = 20e \quad y_p = -20e$$

Q4

$$\theta'' + \theta' - (2\theta = 25 - 12e \quad \theta_p = e - 2$$

$$r = 3, -4$$

$$\theta(t) = C_1 e^{3t} + C_2 e^{-4t} + e - 2$$

Q6

$$y'' - y = -22e + 4$$

$$r^2 - 1 = 0 \quad C_1 e^t + C_2 e^{-t}$$

$$y_p = Ae + B$$

$$y' = A$$

$$y'' = 0$$

$$0 - (Ae + B) = -22e + 4$$

$$A = 22 \quad B = -4$$

$$y(t) = C_1 e^t + C_2 e^{-t} + 22e - 4$$

Q7

$$y'' + 4y' + 5y = 0$$

$$r^2 + 4r + 5 = 0$$

$$16 - 4(5) = -4$$

$$r = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} = -2 \pm i$$

Since we got i our guess for y_p is

$$y'' + 4y' + 5y = 2e^{-2\theta} \cos \theta$$

$$y_p = \theta e^{-2\theta} (A \cos \theta + B \sin \theta)$$

$$y_p = (\theta e^{-2\theta} - 2\theta e^{-2\theta}) (A \cos \theta + B \sin \theta)$$

$$+ \theta e^{-2\theta} (-A \sin \theta + B \cos \theta)$$

$$y_p' = e^{-2\theta} (A \cos \theta + B \sin \theta - 2\theta A \cos \theta - 2\theta B \sin \theta - \theta A \sin \theta + \theta B \cos \theta)$$

$$y_p'' = e^{-2\theta} (A \cos \theta - 2\theta A \cos \theta + \theta B \cos \theta$$

$$+ B \sin \theta - 2\theta B \sin \theta - \theta A \sin \theta)$$

$$y_p' = e^{-2\theta} (A - 2\theta A + \theta B) \cos \theta + (B - 2\theta B - \theta A) \sin \theta$$

$$y_p'' = e^{-2\theta} ((-2A + 4\theta A - 2\theta B) \cos \theta + (-2B + 4\theta B + 2\theta A) \sin \theta$$

$$(-A + 2\theta A - \theta B) \sin \theta + (B - 2\theta B - \theta A) \cos \theta)$$

$$y_p'' = ((-2A + 4\theta A - 2\theta B + B - 2\theta B - \theta A) \cos \theta$$

$$(-2B + 4\theta B + 2\theta A - A + 2\theta A - \theta B) \sin \theta) e^{-2\theta}$$

$$y_p'' = ((3A - 4B) \theta + B - 2A) \cos \theta + (2A + \text{Reorganize}) \sin \theta$$

Q8

$$z'' + z = 7e^{-4x} \quad z(0) = 0, z'(0) = 0$$

$$r^2 + 1 = 0$$

$$z_p = Ae^{-4x}$$

$$r = \pm i$$

$$z_p' = -4Ae^{-4x}$$

$$z_p'' = +16Ae^{-4x}$$

$$16Ae^{-4x} + Ae^{-4x} = 7e^{-4x}$$

$$17Ae^{-4x} = 7e^{-4x}$$

$$A = 7/17$$

$$z(x) = C_1 \cos x + C_2 \sin x + 7/17 e^{-4x}$$

$$z'(x) = -C_1 \sin x + C_2 \cos x - 4/17 e^{-4x}$$

$$C_1 + 7/17 = 0 \quad C_1 = -7/17$$

$$C_2 - 4/17 = 0 \quad C_2 = 4/17$$

$$z(x) = -\frac{7}{17} \sin x + \frac{4}{17} \cos x + \frac{7}{17} e^{-4x}$$

$$z'' + z = ze^{-5x}$$

$$z(x) = C_1 \cos x + C_2 \sin x$$

$$z_p = Ae^{-5x}$$

$$25Ae^{-5x} + Ae^{-5x}$$

$$z_p' = -5Ae^{-5x}$$

$$= 26Ae^{-5x} = 2e^{-5x}$$

$$z_p'' = 25Ae^{-5x}$$

$$A = \frac{1}{13}$$

$$z(x) = C_1 \cos x + C_2 \sin x + \frac{1}{13} e^{-5x}$$

$$z'(x) = -C_1 \sin x + C_2 \cos x - \frac{5}{13} e^{-5x}$$

$$z(0) = C_1 + \frac{1}{13} = 0 \quad C_1 = -\frac{1}{13}$$

$$z'(0) = C_2 - \frac{5}{13} = 0 \quad C_2 = \frac{5}{13}$$

$$z(x) = \frac{1}{13} (5 \sin x - \cos x + e^{-5x})$$

Q9

$$y'' - 4y = 2 \sin 2\theta - 4e^{4\theta}$$

(1) General

$$y'' - 4y = 0$$

$$y_g(\theta) = C_1 e^2 + C_2 e^{-2}$$

$$r^2 - 4 = 0$$

$$r = \pm 2$$

(2) $y'' - 4y = 2 \sin \theta$

$$y_p = A \sin \theta + B \cos \theta$$

$$y' = A \cos \theta - B \sin \theta$$

$$y'' = -A \sin \theta - B \cos \theta$$

$$-A \sin \theta - B \cos \theta + 4A \sin \theta + 4B \cos \theta = 2 \sin \theta$$

$$3A \sin \theta + 3B \cos \theta = 2 \sin \theta$$

$$B = 0 \quad A = 2/3$$

$$y_p = \frac{2}{3} \sin \theta$$

(3) $y'' - 4y = -4e^{4\theta}$

$$16Ae^{4\theta} + 4Ae^{4\theta} =$$

$$y_p = Ae^{4\theta} = 20Ae^{4\theta} = -4e^{4\theta}$$

$$y_p' = 4Ae^{4\theta} \quad 20A = -4$$

$$y_p'' = 16Ae^{4\theta} \quad A = -\frac{1}{5}$$

$$y_p = -\frac{1}{5} e^{4\theta}$$

$$y(\theta) = C_1 e^{\theta} + C_2 e^{-\theta} + \frac{2}{3} \sin \theta - \frac{1}{5} e^{4\theta}$$

$$y'(\theta) = 2C_1 e^{\theta} - 2C_2 e^{-\theta} + \frac{2}{3} \cos \theta - \frac{4}{5} e^{4\theta}$$

$$y(0) = C_1 + C_2 - \frac{1}{5} = 1$$

$$y'(0) = 2C_1 - 2C_2 + \frac{2}{3} - \frac{4}{5} = -1 \quad \frac{9}{5} - \frac{2}{3} = \frac{27-10}{15} = \frac{17}{15}$$

$$C_1 + C_2 = 4/5$$

$$C_1 - C_2 = (1 + 4/5 - 2/3) \cdot 1/2 = \frac{17}{30}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 4/5 \\ 17/30 \end{bmatrix} = \begin{bmatrix} C_1 = \frac{41}{60} \\ C_2 = \frac{7}{60} \end{bmatrix}$$

$$y(\theta) = \frac{41}{60} e^{\theta} + \frac{7}{60} e^{-\theta} + \frac{2}{3} \sin \theta - \frac{1}{5} e^{4\theta}$$

$$y'' - 16y = \sin 4\theta - 2e^{3\theta}; y(0) = 1, y'(0) = -1$$

(1) General

$$r^2 - 16 = 0$$

$$y_g = C_1 e^{4\theta} + C_2 e^{-4\theta}$$

$$r = \pm 4$$

(2) $y'' - 16y = \sin 4\theta - 2e^{3\theta}$

$$y_p = A \sin 4\theta + B \cos 4\theta + C e^{3\theta}$$

$$y'' = -16A \sin 4\theta - 16B \cos 4\theta + 9C e^{3\theta}$$

$$-16A \sin 4\theta - 16B \cos 4\theta + 9C e^{3\theta} - 16A \sin 4\theta - 16B \cos 4\theta - 16C e^{3\theta}$$

$$-32A \sin 4\theta - 32B \cos 4\theta - 7C e^{3\theta} = \sin 4\theta - 2e^{3\theta}$$

$$A = -\frac{1}{32} \quad B = 0 \quad C = \frac{2}{7}$$

$$y(\theta) = -\frac{1}{32} \sin 4\theta + \frac{2}{7} e^{3\theta} + C_1 e^{4\theta} + C_2 e^{-4\theta}$$

$$y' = -\frac{1}{32} \cos 4\theta + \frac{6}{7} e^{3\theta} + 4C_1 e^{4\theta} - 4C_2 e^{-4\theta}$$

$$y(0) = \frac{2}{7} + C_1 + C_2 = 1$$

$$y'(0) = -1/32 + 6/7 + 4C_1 - 4C_2 = -1$$

$$C_1 + C_2 = \frac{5}{7}$$

$$C_1 - C_2 = -109/224$$

$$Qd \quad y'' - 4y = 2 \sin(2\theta) - 4e^{4\theta}$$

$$y(0) = 1$$

$$y'(0) = -1$$

$$r^2 - 4 = 0$$

$$r = \pm 2$$

$$y_h = C_1 e^{2\theta} + C_2 e^{-2\theta}$$

$$y_p = A \sin(2\theta) + B \cos(2\theta) + C e^{4\theta}$$

$$y' = 2A \cos(2\theta) - 2B \sin(2\theta) + 4C e^{4\theta}$$

$$y'' = -4A \sin(2\theta) - 4B \cos(2\theta) + 16C e^{4\theta}$$

$$-4A \sin(2\theta) - 4B \cos(2\theta) + 16C e^{4\theta}$$

$$-4A \sin(2\theta) - 4B \cos(2\theta) - 4C e^{4\theta}$$

$$-8A \sin(2\theta) - 8B \cos(2\theta) + 12C e^{4\theta}$$

$$A = -1/4 \quad B = 0 \quad C = \frac{1}{3}$$

$$y = -\frac{1}{4} \sin(2\theta) + \frac{1}{3} e^{4\theta} + C_1 e^{2\theta} + C_2 e^{-2\theta}$$

$$y' = -\frac{1}{2} \cos(2\theta) + \frac{4}{3} e^{4\theta} + 2C_1 e^{2\theta} - 2C_2 e^{-2\theta}$$

$$y(0) = -\frac{1}{4} + C_1 + C_2 = 1$$

$$y'(0) = -1/2 + 4/3 + 2C_1 - 2C_2 = -1$$

$$C_1 + C_2 = 5/4$$

$$2C_1 - 2C_2 = -7/4$$

$$y'' - 4y = \sin 2\theta - 6e^{5\theta}; y(0) = 1, y'(0) = -1$$

$$y_h = C_1 e^{2\theta} + C_2 e^{-2\theta}$$

$$y_p = A \sin 2\theta + B \cos 2\theta + C e^{5\theta}$$

$$y_p' = 2A \cos 2\theta - 2B \sin 2\theta + 5C e^{5\theta}$$

$$y_p'' = -4A \sin 2\theta - 4B \cos 2\theta + 25C e^{5\theta}$$

$$-4A \sin 2\theta - 4B \cos 2\theta + 25C e^{5\theta}$$

$$-4A \sin 2\theta - 4B \cos 2\theta - 4C e^{5\theta}$$

$$-8A \sin 2\theta - 4B \cos 2\theta + 21C e^{5\theta} = \sin(2\theta) - 6e^{5\theta}$$

$$A = -\frac{1}{8} \quad B = 0 \quad C = -\frac{2}{7}$$

$$y = -\frac{1}{8} \sin 2\theta - \frac{2}{7} e^{5\theta} + C_1 e^{2\theta} + C_2 e^{-2\theta}$$

$$y(0) = -\frac{1}{8} - \frac{2}{7} + C_1 + C_2 = 1$$

$$C_1 + C_2 = 9/7$$

$$y'(0) = -\frac{1}{4} \cos 2\theta - \frac{10}{7} e^0 + 2C_1 - 2C_2 = -1$$

$$-\frac{1}{4} - \frac{10}{7} + 2C_1 - 2C_2 = -1$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 9/7 \\ 19/14 \end{bmatrix}$$

$$y = -\frac{1}{8} \sin 2\theta - \frac{2}{7} e^{5\theta} + \frac{13}{16} e^{2\theta} + \frac{53}{112} e^{-2\theta}$$

Q10

$$y'' - y = e^{6t} + 7te^{6t} + 5t^2 e^{6t}$$

$$y' - y = (1 + 7t + 5t^2) e^{6t}$$

$$Q_1 \quad y'' + y = 3 \sec t$$

$$r^2 + 1 = 0$$

$$r = \pm i$$

$$y_1 = C_1 \cos t \quad y_2 = C_2 \sin t$$

$$y_1' = -\sin t \quad y_2' = \cos t$$

$$\begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t + \sin^2 t = 1$$

$$v_1 = \int \frac{3 \sec t \cdot \sin t}{\cos^2 t + \sin^2 t} dt = \int 3 \sec t \cdot \sin t dt$$

$$= 3 \int \tan t dt = 3 \ln |\cos t|$$

$$v_2 = \int \frac{3 \sec t \cdot \cos t}{\cos^2 t + \sin^2 t} dt = \int 3 dt = 3t$$

$$= 3 \cos t \ln |\cos t| + 3t \sin t + C_1 \cos t + C_2 \sin t$$

$$Q_2 \quad y'' + 8y' + 16y = 5e^{-4t}$$

$$r^2 + 8r + 16 = 0$$

$$(r+4)^2 = 0$$

$$r = -4$$

$$y_p = C_1 e^{-4t} + t C_2 e^{-4t}$$

$$y_1 = e^{-4t} \quad y_2 = t e^{-4t}$$

$$y_1' = -4e^{-4t} \quad y_2' = (1-4t)e^{-4t}$$

$$(1-4t)e^{-8t} + 4te^{-8t} = e^{-8t}$$

$$v_1 = \int \frac{5e^{-4t} \cdot te^{-4t}}{e^{-8t}} dt = \int 5t dt = \frac{5}{2} t^2$$

$$v_2 = \int \frac{e^{-4t} \cdot 5e^{-4t}}{e^{-8t}} dt = \int 5 dt = 5t$$

$$\frac{5}{2} t^2 e^{-4t} + 5t^2 e^{-4t}$$

$$y'' + 10y' + 25y = 4e^{-5t}$$

$$r^2 + 10r + 25 = 0 \quad y_p = v_1 e^{-5t} + v_2 t e^{-5t}$$

$$r = -5$$

$$y_1 = e^{-5t} \quad y_2 = t e^{-5t}$$

$$y_1' = -5e^{-5t} \quad y_2' = (1-5t)e^{-5t}$$

$$v_1' e^{-5t} + v_2' t e^{-5t} = 0$$

$$-5v_1' e^{-5t} + (1-5t)v_2' = 4e^{-4t}$$

$$v_1' = -v_2' t e^{-5t}$$

$$8v_2' e^{-5t} + (1-5t)e^{-5t} v_2' = 4e^{-4t}$$

$$v_2' (1 - 5t) = 4e^{-4t}$$

$$v_2' = 4$$

$$v_1' = -4t e^{-5t}$$

$$v_2 = 4t$$

$$u = t \quad dv = e^{-5t}$$

$$du = 1 \quad v = -\frac{1}{5} e^{-5t}$$

$$v_1 = -4 \int t e^{-5t} dt = -4 \left(-\frac{1}{5} t e^{-5t} + \frac{1}{5} \int e^{-5t} dt \right)$$

$$v_1 = \frac{4}{5} t e^{-5t} + \frac{4}{25} e^{-5t}$$

$$y = 4t e^{-5t} + \frac{4}{5} t e^{-5t} + \frac{4}{25} e^{-5t} + C_1 e^{-5t} + t C_2 e^{-5t}$$

$$v_1' e^{-5t} + v_2' t e^{-5t} = 0$$

$$-v_1' e^{-5t} + (1-t)v_2' = 4e^{-4t}$$

$$v_1' = -6v_2'$$

$$t v_2' + (1-t)v_2' = 4$$

$$v_2' = 4$$

$$v_1' = -4t$$

$$v_2' = 4$$

$$v_1 = -2t^2$$

$$v_2 = 4t$$

$$y = -2t^2 e^{-t} + 4t^2 e^{-t} + C_1 e^{-t} + t C_2 e^{-t}$$

$$y = 2t^2 e^{-t} + C_1 e^{-t} + t C_2 e^{-t}$$

$$Q_3 \quad y'' + y = \csc^2 t$$

$$r^2 + 1 = 0$$

$$y_1 = \sin t$$

$$y_2 = \cos t$$

$$r = \pm i$$

$$y_1' = \cos t$$

$$y_2' = -\sin t$$

$$\begin{vmatrix} \sin t & \cos t \\ \cos t & -\sin t \end{vmatrix} = -1$$

$$v_1 = \int \csc^2 t \cdot \sin t dt = \int \csc t dt = -\ln |\csc t| - \cot t$$

$$v_2 = \int \csc^2 t \cdot \cos t dt = \int \frac{\cos t}{\sin^2 t} dt = \int \frac{1}{u^2} du = -\frac{1}{\sin t}$$

$$y = -\sin t \ln |\csc t| - \cot t - \cot t + C_1 \sin t + C_2 \cos t$$

$$y'' + y = \csc^2 t$$

$$r^2 + 1 = 0$$

$$y_1 = \cos t$$

$$y_2 = \sin t$$

$$r = \pm i$$

$$y_1' = -\sin t$$

$$y_2' = \cos t$$

$$\begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = 1$$

$$v_2 = \int \csc^2 t \cdot \cos t dt = \int \frac{\cos t}{\sin^2 t} dt = \int \frac{1}{u^2} du = -\frac{1}{\sin t}$$

$$v_1 = -\int \csc^2 t \cdot \sin t dt = -\int \csc t dt = +\ln |\csc t| + \cot t$$

$$y = \ln |\cot t + \csc t| - \frac{1}{\sin t} + C_1 e^{-t} + C_2 e^{-t}$$

$$y'' + 25y = 5 \csc^5 t$$

$$r = \pm 5i$$

$$y_1 = \cos 5t$$

$$y_2 = \sin 5t$$

$$y_1' = -5 \sin 5t$$

$$y_2' = 5 \cos 5t$$

$$\begin{vmatrix} \cos 5t & \sin 5t \\ -5 \sin 5t & 5 \cos 5t \end{vmatrix} = 5 \cos^2 5t + 5 \sin^2 5t = 5$$

$$v_1 = -\int \frac{5 \csc^5 t \cdot \sin 5t}{5} dt = -\int \csc^4 t \cdot \sin 5t dt = -\int \csc^4 t dt = -\frac{1}{5} \ln |\csc t| + \cot t$$

$$\frac{1}{5} \ln |\csc t| + \cot t = v_1$$

$$v_2 = \int \frac{5 \csc^5 t \cdot \cos 5t}{5} dt = \int \frac{\cos 5t}{\sin^5 t} dt = \frac{1}{5} \int \frac{1}{u^5} du$$

$$= -\frac{1}{5} \csc t$$

$$y = \frac{1}{5} \cos 5t \ln |\csc 5t| + \cot 5t - \frac{1}{5} + C_1 \cos 5t + C_2 \sin 5t$$

$$Q_4 \quad y'' + y = 5 \sec^3 \theta$$

$$r = \pm i$$

$$y_1 = \cos \theta$$

$$y_2 = \sin \theta$$

$$y_1' = -\sin \theta$$

$$y_2' = \cos \theta$$

$$v_1 = -\int \frac{5 \sec^3 \theta \cdot \sin \theta}{5} d\theta = -\int \frac{\sin \theta}{\cos^3 \theta} d\theta = -\frac{1}{2} \sec^2 \theta$$

$$v_2 = \int \frac{5 \sec^3 \theta \cdot \cos \theta}{5} d\theta = \int \sec^2 \theta d\theta = \tan \theta$$

$$= -\frac{1}{2} \sec^2 \theta + \sin \theta \tan \theta + C_1 \cos \theta + C_2 \sin \theta$$

$$y'' + 16y = 3 \sec^3 4\theta$$

$$r^2 + 16 = 0$$

$$y_1 = \cos 4\theta$$

$$y_2 = \sin 4\theta$$

$$r = \pm 4i$$

$$y_1' = -4 \sin 4\theta$$

$$y_2' = 4 \cos 4\theta$$

$$\begin{vmatrix} \cos 4\theta & \sin 4\theta \\ -4 \sin 4\theta & 4 \cos 4\theta \end{vmatrix} = 4 \cos^2 4\theta + 4 \sin^2 4\theta = 4$$

$$v_1 = -\int \frac{3 \sec^3 4\theta \cdot \sin 4\theta}{4} d\theta = -\frac{3}{4} \int \tan 4\theta \cdot \sec^2 4\theta d\theta = \frac{3}{16} \int u du$$

$$= -\frac{3}{32} \tan^2 4\theta$$

$$v_2 = \int \frac{3 \sec^3 4\theta \cdot \cos 4\theta}{4} d\theta = \frac{3}{4} \int \sec 4\theta d\theta = \frac{3}{16} \tan \theta$$

$$= -\frac{3}{32} \tan^2 4\theta \cdot \cos 4\theta + \frac{3}{16} \tan 4\theta \cdot \sin 4\theta$$

$$= -\frac{3}{32} \tan 4\theta \sin 4\theta + \frac{3}{16} \tan 4\theta \cdot \sin 4\theta$$

$$\frac{3}{32} \tan 4\theta \sin 4\theta$$

$$\frac{1}{6} y'' + 6y = 3 \tan 6t - \frac{1}{4} e^{3t}$$

$$\frac{1}{6} r^2 + 6 = 0$$

$$y_1 = \cos 6t$$

$$y_2 = \sin 6t$$

$$r^2 + 36 = 0$$

$$y_1' = -6 \sin 6t$$

$$y_2' = 6 \cos 6t$$

$$r = \pm 6i$$

$$\begin{vmatrix} \cos 6t & \sin 6t \\ -6 \sin 6t & 6 \cos 6t \end{vmatrix} = 6$$

$$v_1 = -\int \frac{3 \tan 6t \cdot \sin 6t}{6} dt = -\frac{1}{2} \int \frac{\sin^2 6t}{\cos 6t} dt$$

$$= -\frac{1}{2} \int \sec 6t - \cos 6t dt =$$

$$= -\frac{1}{12} \ln |\sec 6t| + \tan 6t - \frac{1}{12} \sin 6t$$

$$v_2 = \int \frac{3 \tan 6t \cdot \cos 6t}{6} dt = \frac{1}{2} \int \sin 6t dt = -\frac{1}{12} \cos 6t$$

$$y = \frac{1}{12} (\cos 6t (\ln |\sec 6t| + \tan 6t) + \sin 6t) + \cos 6t \sin 6t$$

$$\frac{1}{6} y'' + 6y = -\frac{1}{4} e^{3t}$$

$$A e^{3t}$$

$$3A e^{2t}$$

$$9A e^{3t}$$

$$\frac{3}{2} A e^{3t} + 6A e^{3t} = -\frac{1}{4} e^{3t}$$

$$\frac{15}{2} A e^{3t} = -\frac{1}{4} e^{3t}$$

$$A = -\frac{1}{30}$$

$$y_p = -\frac{1}{30} e^{3t}$$

$$y = -\frac{1}{12} \cos 6t (\ln |\sec 6t| + \tan 6t) + \frac{1}{12} \sin 6t + C_1 \cos 6t + C_2 \sin 6t$$

$$\frac{1}{4} y'' + 4y = \tan 4t - \frac{1}{2} e^t$$

$$\textcircled{1} \quad \frac{1}{4} y'' + 4y = \tan 4t$$

$$\frac{1}{4} r^2 + 4 = 0$$

$$r^2 + 16 = 0$$

$$y_1 = \cos 4t$$

$$y_2 = \sin 4t$$

$$r = \pm 4i$$

$$y_1' = -4 \sin 4t$$

$$y_2' = 4 \cos 4t$$

$$\begin{vmatrix} \cos 4t & \sin 4t \\ -4 \sin 4t & 4 \cos 4t \end{vmatrix} = 4$$

$$v_1 = -\int \frac{\tan 4t \cdot \sin 4t}{4} dt$$

$$= -\frac{1}{4} \int \frac{\sin^2 t}{\cos t} dt = -\frac{1}{4} \int \sec 4t - \cos 4t dt$$

$$= -\frac{1}{4} \left(\frac{1}{4} \ln |\sec 4t| + \tan 4t - \frac{1}{4} \sin 4t \right)$$

$$= -\frac{1}{16} (\ln |\sec 4t| + \tan 4t) - \frac{1}{16} \sin 4t$$

$$v_2 = \int \frac{\tan 4t \cdot \cos 4t}{4} dt = \frac{1}{4} \int \sin 4t dt = -\frac{1}{16} \cos 4t$$

$$y = -\frac{1}{16} \cos 4t (\ln |\sec 4t| + \tan 4t) + \frac{1}{16} \sin 4t + C_1 \cos 4t + C_2 \sin 4t$$

$$\textcircled{2} \quad \frac{1}{4} y'' + 4y = -\frac{1}{2} e^t$$

$$\frac{1}{4} A e^t + 4A e^t = -\frac{1}{2} e^t$$

$$\frac{17}{4} A = -\frac{1}{2}$$

$$A = -\frac{1}{17} \cdot \frac{4}{17} = -\frac{2}{17}$$

$$y = -\frac{2}{17} e^t$$

$$y = -\frac{1}{16} \cos 4t (\ln |\sec 4t| + \tan 4t) + \frac{1}{16} \sin 4t + C_1 \cos 4t + C_2 \sin 4t$$

$$\textcircled{6} \quad 6y'' - (6+1)y' + y = 11e^t$$

$$D = \frac{d}{dt} \quad y'' + y - 2 = 0$$

$$D^2 y + D y - 2y = 0$$

$$(D^2 + D - 2)[y] = 0$$

$$\begin{array}{l|l} (y' - x) + y = 0 & x' + 3y = 0 \\ 4x(y' - y) = 0 & x' - y' = 0 \\ (D-1)[x] + y = 0 & D[x] + 3y = 0 \\ 4x + (D-1)[y] = 0 & D[x] - D[y] = 0 \\ & -3y - D[y] = 0 \end{array}$$

$$D(D[x] + 3y) = 0$$

$$3(D[x] - D[y]) = 0 \quad (D+3)[y] = 0$$

$$y' + 3y = 0$$

$$D^2[x] + 3D[y] = 0 \quad y = C e^{-3t}$$

$$3D[x] - 3D[y] = 0$$

$$D^2[x] + 3D[x] = 0$$

$$x'' + 3x' = 0 \quad C_1 e^{0t} + C_2 e^{-3t}$$

$$a^2 + 3a = a(a+3)$$

$$(2) \quad x' = 3x - 2y + \sin t$$

$$y' = 4x - y - \cos t$$

$$x' - 3x + 2y = \sin t$$

$$-y' + 4x - y = \cos t$$

$$D(x) - 3x + 2y = \sin t$$

$$-D(y) + 4x - y = \cos t$$

$$(D-3)x + 2y = \sin t$$

$$4x - (D-1)y = \cos t$$

$$\uparrow (D-1)(D-3)x + 2(D-1)y = \sin t (D-1)$$

$$8x - 2(D-1)y = 2\cos t$$

$$(D-1)(D-3)x + 8x = \sin t (D-1) + 2\cos t$$

$$(D-1)(D-3) + 8)x = 0 \sin t - \sin t + 2\cos t$$

$$(D^2 - 3D - D + 3 + 8)x$$

$$x'' - 4x' + 11x = -\sin t - \cos t$$

$$r^2 - 4r + 11 = 0$$

$$r = \frac{4 \pm \sqrt{16 - 4 \cdot 11}}{2} = \frac{4 \pm 2\sqrt{7}i}{2} = 2 \pm \sqrt{7}i$$

$$x_p(t) = e^{2t} (C_1 \cos(\sqrt{7}t) + C_2 \sin(\sqrt{7}t))$$

$$A \sin t + B \cos t = x(t)$$

$$-A \cos t + B \sin t = x'(t)$$

$$-A \sin t - B \cos t = x'(t)$$

$$-A \sin t - B \cos t + 4A \cos t - 4B \sin t + 11A \sin t + 11B \cos t =$$

$$10A \sin t + 10 \cos t + 4A \cos t - 4B \sin t =$$

$$(10A - 4B) \sin t + (10B + 4A) \cos t$$

$$\begin{bmatrix} 10 & -4 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad A = -\frac{1}{10}$$

$$B = 0$$

$$x(t) =$$

$$x' = 3x - 2y + \sin t$$

$$y' = 4x - y - \cos t$$

$$x' - 3x - 2y = \sin t$$

$$y' - 4x + y = -\cos t$$

$$(D-3)x - 2y = \sin t$$

$$(D-1)y - 4x = -\cos t$$

$$4(D-3)x - 8y = 4\sin t$$

$$(D-1)(D-3)y - 4(D-3)x = (3-D)\cos t$$

$$(D-1)(D-3)y - 8y = 4\sin t + (3-D)\cos t$$

$$(D^2 - 4D - 5)y = 4\sin t + 3\cos t + \sin t$$

$$y' - 4y' - 5y = 3\sin t + 3\cos t$$

$$r^2 - 4r - 5 = (r+1)(r-5)$$

$$y_g(t) = C_1 e^{-4t} + C_2 e^t$$

$$y_p(t) = A \sin t + B \cos t$$

$$y'_p = -A \cos t + B \sin t$$

$$y''_p = -A \sin t - B \cos t$$

$$-A \sin t - B \cos t + 4A \cos t - 4B \sin t - 5A \sin t - 5B \cos t$$

$$= -6A \sin t - 6 \cos t + 4A \cos t - 4B \sin t$$

$$\begin{bmatrix} -6 & -4 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad A = -\frac{1}{2}$$

$$B = 0$$

$$y(t) = C_1 e^{-4t} + C_2 e^t - \frac{1}{2}$$

$$(3) \quad 2x' + y' - 4x - 6y = e^{-3t}$$

$$x' + y' + 8x + 3y = e^{6t}$$

$$2x' - 4x + y' - 6y = e^{-3t}$$

$$x' + 8x + y' + 3y = e^{6t}$$

$$(2D-4)[x] + (D-6)[y] = e^{-3t}$$

$$(D+8)[x] + (D+3)[y] = e^{6t}$$

$$\uparrow (2D-4)(D+3)x + (D-6)(D+3)y = (D+3)e^{-3t}$$

$$\uparrow (D+8)(D-6)x + (D+3)(D-6)y = e^{6t} (D-6)$$

$$((2D-4)(D+3) - (D+8)(D-6))x = (D+3)e^{-3t} - e^{6t}(D-6)$$

$$x'' + 36x = D e^{-3t} + 3 e^{-3t} - D e^{6t} + 6 e^{6t}$$

$$= -3 e^{-3t} + 3 e^{-3t} - 6 e^{6t} + 6 e^{6t}$$

$$x'' + 36x = 0$$

$$(2D-4)[x] + (D-6)[y] = e^{-3t}$$

$$(D+8)[x] + (D+3)[y] = e^{6t}$$

$$(2D-4)(D+8)x + (D+8)(D-6)y = (D+8)e^{-3t}$$

$$(D+8)(2D-4)x + (D+3)(2D-4)y = (2D-4)e^{6t}$$

$$(D+8)(D-6)y - (D+3)(2D-4)y =$$

$$(-D^2 - 36)y = -3e^{-3t} + 8e^{-3t} + 12D e^{6t} - 4e^{6t}$$

$$-y'' - 36y = 5e^{-3t} + 8e^{6t}$$

$$-y'' - 36y = 0$$

$$C_1 \cos 6t + C_2 \sin 6t = y_g$$

$$y_p = A e^{-3t} + B e^{6t}$$

$$y'_p = -3A e^{-3t} + 6B e^{6t}$$

$$y''_p = 9A e^{-3t} + 36B e^{6t}$$

$$-9A e^{-3t} - 36B e^{6t} - 36A e^{-3t} - 36B e^{6t}$$

$$-45A e^{-3t} - 72B e^{6t}$$

$$-45A = 5 \quad -72B = 8$$

$$A = -\frac{1}{9}$$

$$B = -\frac{1}{9}$$

$$y = C_1 \cos 6t + C_2 \sin 6t - \frac{1}{9} e^{-3t} - \frac{1}{9} e^{6t}$$

$$2x' + y' - 4x - 7y = e^{-3t}$$

$$x' + y' + 4x + 3y = e^{7t}$$

$$(2D-4)[x] + (D-7)[y] = e^{-3t}$$

$$(D+4)[x] + (D+3)[y] = e^{7t}$$

$$L_1 = 2D-4, \quad L_2 = D-7$$

$$L_3 = D+4, \quad L_4 = D+3$$

$$(L_1 L_4 - L_2 L_3)[x] = g_1$$

$$g_1 = L_4 f_1 - L_2 f_2$$

$$(L_1 L_4 - L_2 L_3)[y] = g_2$$

$$g_2 = L_1 f_2 - L_3 f_1$$

$$L_1 L_4 - L_2 L_3 = (2D-4)(D+3) - (D+4)(D-7)$$

$$= D^2 + 1$$

$$C_1 \cos t + C_2 \sin t \rightarrow \text{general}$$

$$g_1 = (D+3)e^{-3t} - (D-7)e^{7t}$$

$$g_1 = -3e^{-3t} + 3e^{3t} - 7e^{7t} + 7e^{7t}$$

$$g_1 = 0$$

$$x(t) = C_1 \cos t + C_2 \sin t$$

$$g_2 = (2D-4)e^{7t} - (D+4)e^{-3t}$$

$$14e^{7t} - 4e^{7t} + 3e^{-3t} - 4e^{-3t}$$

$$g_2 = 10e^{7t} - e^{-3t}$$

$$g_2 = \frac{1}{10} e^{7t} - \frac{1}{10} e^{-3t}$$

$$(2D-4)[x] + (D-7)[y] = 0$$

$$(D+4)[x] + (D+3)[y] = 0$$

$$-2C_1 \sin t + 2C_2 \cos t - 4C_1 \cos t - 4C_2 \sin t$$

$$-C_3 \sin t + C_4 \cos t - 7C_3 \cos t - 7C_4 \sin t$$

$$(-4C_1 + 2C_2 - 7C_3 + 4C_4) \cos t + (-2C_1 - 4C_2 - C_3 - 7C_4) \sin t$$

$$-C_1 \sin t + C_2 \cos t + 4C_1 \cos t + 4C_2 \sin t$$

$$-C_3 \sin t + C_4 \cos t + 3C_3 \cos t + 3C_4 \sin t$$

$$(4C_1 + C_2 + 3C_3 + 4C_4) \cos t + (-C_1 + 4C_2 - C_3 + 3C_4) \sin t$$

$$-4C_1 + 2C_2 - 7C_3 + 4C_4$$

$$-2C_1 - 4C_2 - C_3 - 7C_4$$

$$4C_1 + C_2 + 3C_3 + 4C_4$$

$$-C_1 + 4C_2 - C_3 + 3C_4$$

$$-5C_1 + 3C_2 - 4C_3 + 2C_4$$

$$-3C_1 - 5C_2 - 2C_3 - 4C_4$$

$$-5C_1 + 3C_2 - 4C_3 + 2C_4 = 0$$

$$-3C_1 - 5C_2 - 2C_3 - 4C_4 = 0$$

$$4C_3 = -5C_1 + 3C_2 + 2C_4$$

$$4C_4 = -3C_1 - 5C_2 - 2C_3$$

$$4C_3 = -5C_1 + 3C_2 + 2(-3C_1 - 5C_2 - 2C_3)$$

$$4C_3 = -5C_1 + 3C_2 - 6C_1 - 10C_2 - 4C_3$$

$$4C_3 = -11C_1 - 7C_2$$

$$8C_3 = -11C_1 - 7C_2$$

$$C_3 = -\frac{11C_1 + 7C_2}{8}$$

$$4C_4 = -3C_1 - 5C_2 + \frac{11C_1 + 7C_2}{4}$$

$$4C_4 = \frac{-12C_1 - 20C_2 + 11C_1 + 7C_2}{4} = \frac{-C_1 - 13C_2}{4}$$

$$Dx + 2x - Dy = e^{5t}$$

$$8x + D^2 y = 0$$

$$D(D+2)x + D^2 y = D e^{5t}$$

$$-8x - D^2 y = 0$$

$$(D(D+2) - 8)[x] = D e^{5t}$$

$$(D^2 - 6D)[x] = D e^{5t}$$

$$r^2 - 6r = 0$$

$$r(r-6) = 0 \quad x(t) = C_1 + C_2 e^{6t}$$

$$x'' - 6x' = 5e^{5t}$$

$$x_p = A e^{5t}$$

$$x' = 5A e^{5t} \quad 25A e^{5t} - 30A e^{5t}$$

$$A = -1$$

$$x'' = 25A e^{5t}$$

$$x(t) = C_1 + C_2 e^{6t} - e^{5t}$$

$$(D+2)[x] + D[y] = e^{5t}$$

$$8x + D^2[y] = 0$$

$$8D[y] - (D+2)D^2[y] = 8e^{5t}$$

$$(8D - D^3 + 2D^2)y = 8e^{5t}$$

$$-r^3 + 2r^2 + 8r = 0$$

$$r(r^2 - 2r - 8) = 0$$

$$r(r-4)(r+2) = 0$$

$$C_1 + C_2 e^{4t} + C_3 e^{-2t} = x_g(t)$$

$$-125A e^{5t} + 50A e^{5t} + 40A e^{5t} = -35A e^{5t}$$

$$-35A = 8$$

$$A = -\frac{8}{35}$$

$$xy''' - 8y' + e^x y = x^3 - 4$$

$$y(-8) = 1, y'(-8) = 0, y''(-8) = 2$$

$$y''' - \frac{8y'}{x} + \frac{e^x y}{x}$$

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$$3xy''' - 12y'' = -48, x > 0$$

$$y(1) = 4, y'(1) = 2, y''(1) = -86$$

$$y_p = 2x^2; \{1, x, x^6\}$$

$$y''' - \frac{12y''}{3x} = -48$$

$$y(x) = 2x^2 + C_1 + C_2 x + C_3 x^6$$

$$2 + C_1 + C_2 + C_3 = 4$$

$$C_1 + C_2 + C_3 = 2$$

$$y'(x) = 4x + C_2 + 6C_3 x^5$$

$$4 + C_2 + 6C_3 = 2$$

$$C_2 + 6C_3 = -2$$

$$y''(x) = 4 + 30C_3 x^4$$

$$4 + 30C_3 = -86$$

$$30C_3 = -90$$

$$C_1 = -11 \quad C_2 = 16 \quad C_3 = -3$$

$$3xy''' - 12y'' = -48 \quad x > 0$$

$$y(1) = 4, y'(1) = 2, y''(1) = -86$$

$$y_p = 2x^2; \{1, x, x^6\}$$

$$y(x) = 2x^2 + C_1 + C_2 x + C_3 x^6$$

$$2 + C_1 + C_2 + C_3 = 4$$

$$C_1 + C_2 + C_3 = 2$$

$$y'(x) = 4x + C_2 + 6C_3 x^5$$

$$4 + C_2 + 6C_3 = 2$$

$$C_2 + 6C_3 = -2$$

$$y''(x) = 4 + 30x^4 C_3$$

$$4 + 30C_3 = -86$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 2 \\ 0 & 1 & 6 & | & -2 \\ 0 & 0 & 30 & | & 90 \end{bmatrix} \quad \begin{matrix} C_1 = \\ C_2 = \\ C_3 = \end{matrix}$$

$$y_p(x) = 4x^2 + C_1 + C_2 x + C_3 x^5$$

$$4 + C_1 + C_2 + C_3 = 2$$

$$C_1 + C_2 + C_3 = -2$$

$$y'_p(x) = 8x + C_2 + 5x^4 C_3$$

$$8 + C_2 + 5C_3 = -5$$

$$C_2 + 5C_3 = -13$$

$$y''(x) = 8 + 20x^3 C_3$$

$$8 + 20C_3 = -32$$

$$20C_3 = -40$$

$$\begin{bmatrix} 1 & 1 & 1 & | & -2 \\ 0 & 1 & 5 & | & -3 \\ 0 & 0 & 20 & | & -40 \end{bmatrix}$$

$$y(x) = 4x^2 + 3 - 3x - 2x^5$$

$$\textcircled{1} \quad y''' - 2y'' - 35y' = 0$$

$$r^3 - 2r^2 - 35r = 0$$

$$r(r^2 - 2r - 35) = 0$$

$$r(r-7)(r+5) = 0$$

$$r = \{-5, 0, 7\}$$

$$y_g = C_1 e^{-5x} + C_2 + C_3 e^{7x}$$

$$\textcircled{2} \quad y''' - 2y'' - 21y' - 18y = 0$$

$$r^3 - 2r^2 - 21r - 18 = 0$$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -21 & -18 \\ & \downarrow & -1 & 3 & 18 \\ \hline & 1 & -3 & -18 & 0 & (r+1) \end{array}$$

$$(r^2 - 3r - 18)(r+1) = 0$$

$$(r-6)(r+3)(r+1) = 0$$

$$\begin{array}{c} 18 \\ \swarrow \searrow \\ 2 \quad 3 \quad 3 \end{array}$$

$$y_g(x) = C_1 e^{-3x} + C_2 e^{-x} + C_3 e^{6x}$$

$$\textcircled{3} \quad y^4 + 4y'' + 4y = 0$$

$$r^4 + 4r^2 + 4 = 0$$

$$(r^2 + 2)^2 = 0$$

$$r^2 + 2 = 0$$

$$r^2 = -2$$

$$r = \pm \sqrt{2} i$$

4 roots

$$y_g = C_1 \cos \sqrt{2}x + C_2 x \cos \sqrt{2}x + C_3 \sin \sqrt{2}x + C_4 x \sin \sqrt{2}x$$

$$\textcircled{1} y'' + 9y' - 10y = x e^x + 5$$

$$\textcircled{2} y''' + y'' - 16y' + 20y = 8e^{-2x} + \cos x$$

$$r^3 + r^2 - 16r + 20 = 0$$

$$\begin{array}{r|rrrr} -5 & 1 & 1 & -16 & 20 \\ & \downarrow & -5 & 20 & -20 \\ \hline & 1 & -4 & 4 & 0 \end{array} \quad (r+5)$$

$$(r^2 - 4r + 4)(r+5) = 0$$

$$(r-2)^2(r+5) = 0$$

$$y_p = C_1 e^{-5x} + C_2 e^{2x} + C_3 x e^{2x}$$

$$A e^{-2x} + B \sin x + C \cos x = y$$

$$-2A e^{-2x} + B \cos x - C \sin x = y'$$

$$4A e^{-2x} - B \sin x - C \cos x = y''$$

$$-8A e^{-2x} - B \cos x + C \sin x = y'''$$

$$-8A e^{-2x} - B \cos x + C \sin x$$

$$4A e^{-2x} - B \sin x - C \cos x$$

$$-16(-2A e^{-2x} + B \cos x - C \sin x)$$

$$20(A e^{-2x} + B \sin x + C \cos x)$$

$$-8A e^{-2x} - B \cos x + C \sin x$$

$$4A e^{-2x} - B \sin x - C \cos x$$

$$32A e^{-2x} - 16B \cos x + 16C \sin x$$

$$20A e^{-2x} + 20B \sin x + 20C \cos x$$

$$48A e^{-2x} + (19B + 17C) \sin x + (-17B - 19C) \cos x$$

$$48A = 8 \quad \begin{bmatrix} 19 & 17 \\ -17 & -19 \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$$

$$A = \frac{1}{6}$$

$$y_p = \frac{1}{6} A e^{-2x} + \frac{17}{72} \sin x - \frac{19}{72} \cos x$$

$$\textcircled{3} y''' - 6y'' + 9y' - 4y = 4e^{-x} + \sin x$$

$$y''' - 6y'' + 9y' - 4y = 0$$

$$[D^3 - 6D^2 + 9D - 4]y = 0$$

$$\begin{array}{r|rrrr} 1 & 1 & -6 & 9 & -4 \\ & \downarrow & 1 & -5 & 4 \\ \hline & 1 & -5 & 4 & 0 \end{array} \quad (D-1)$$

$$(D^2 - 5D + 4)(D-1) = 0$$

$$(D-4)(D-1)^2 = 0$$

$$A[4e^{-x} + \sin x] = 0$$

$$(D+1)(D^2+1)$$

$$(D-4)(D-1)^2(D+1)(D^2+1)[y] = 0$$

$$y(x) = C_1 e^{4x} + C_2 e^x + C_3 x e^x + C_4 e^{-x} + C_5 \sin x + C_6 \cos x$$

$$A e^x + B \cos x + C \sin x = y$$

$$-A e^x - B \sin x + C \cos x = y'$$

$$A e^x - B \cos x - C \sin x = y''$$

$$-A e^x + B \sin x - C \cos x = y'''$$

$$-A e^x + B \sin x - C \cos x$$

$$-6(A e^x - B \cos x - C \sin x)$$

$$9(-A e^x - B \sin x + C \cos x)$$

$$-4(A e^x + B \cos x + C \sin x)$$

$$-A e^x + B \sin x - C \cos x$$

$$-6A e^x + 6B \cos x + 6C \sin x$$

$$-9A e^x - 9B \sin x + 9C \cos x$$

$$-4A e^x - 4B \cos x - 4C \sin x$$

$$-20A e^x - (8B + 2C) \sin x + (2B + 8C) \cos x$$

$$-20A = 4 \quad \begin{bmatrix} -8 & 2 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$A = -\frac{1}{20}$$

$$\textcircled{4} y''' - 9y'' + 27y' - 27y = 9e^{3x}$$

$$r^3 - 9r^2 + 27r - 27 = 0$$

$$(r-3)^3 = 0$$

$$(C_1 + C_2 x + C_3 x^2) e^{3x}$$

$$A[9e^{3x}] = (D-3)$$

$$(D-3)^3(D-3) = (D-3)^4$$

$$A e^{3x} = y$$

$$3A e^{3x} = y'$$

$$9A e^{3x} = y''$$

$$27A e^{3x} = y'''$$

$$27A - 9(9A) + 27(3A) - 27A$$

$$F(s) = \int_0^{\infty} e^{-st} (-6t^2) dt$$

$$dv = e^{-st} \quad u = t^2$$

$$v = -\frac{1}{s} e^{-st} \quad du = 2t$$

$$\lim_{b \rightarrow \infty} \left[-6 \left(-\frac{1}{s} t^2 e^{-st} + \frac{2}{s} \int_0^b e^{-st} t dt \right) \right]$$

$$dv = e^{-st} \quad u = t$$

$$v = -\frac{1}{s} e^{-st} \quad du = 1$$

$$\lim_{b \rightarrow \infty} \left[-6 \left[\frac{1}{s} (-t^2 e^{-st} + 2 \left(-\frac{1}{s} (-t e^{-st} + \int e^{-st} dt) \right)) \right] \right]$$

$$\mathcal{L}\{3e^{-at}\} = 3 \frac{1}{s+a}$$

$$\mathcal{L}\{t^3\} = \frac{3!}{s^4} = \frac{6}{s^4}$$

$$\mathcal{L}\{4t\} = 4 \frac{1}{s^2} = \frac{4}{s^2}$$

$$\mathcal{L}\{2\} = \frac{2}{s}$$