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Assingnment 1

Section 1.1

① Which of these sentences are propositions? What are the truth values of those propositions?

ⓐ Boston is the capital of Massachusetts.

Proposition, True

ⓑ Maiami is the capital of Florida.

Proposition, False

ⓒ $2 + 3 = 5$

Proposition, True

ⓓ $5 + 7 = 10$

Proposition, False

ⓔ $x + 2 = 11$

Sentence

ⓕ Answer this question.

what question?

Sentence

Note! Final answers will be highlyted pink
saves me time

⑦ What is the negaion of each of these propostions?

ⓐ Steve has more than 100 gb disk space on his laptop

Steve less than or equal to 100 gb disk space on his laptop

ⓑ Zack blocks emails and texts from Jennifer

Zack accepts all emails and texts from Jennifer

ⓒ $7 \cdot 11 \cdot 13 = 999$

$7 \cdot 11 \cdot 13 \neq 999$

ⓓ Diane rode her bicycle 100 mi on Sunday

Diane did not rode her bicycle 100 mi on Sunday

⑬ Let p and q be the propositions

p: It is bellow feezing

q: It is snowing

Note! But ⇒ ∧

write these propositions using p and q using logical connectives (incluiding negations)

ⓑ It is bellow freezing but not snowing

$p \vee \neg q$

ⓕ Either it is below freezeng, or it is snowing, but it is bellow freezen if it is not snowing

$(p \vee q) \vee (q \rightarrow p)$

⑰ Let p, q and r be the positions

p : Grizly bears have been seen in the area
q: It's safe to hike on the trail
r: Berries are ripe on the trail

Write propositions using p,q and r using logical connective

ⓓ Berries are ripe along the trail, but grizzly bears have not been seen in the area.

$r \vee \neg p$

ⓔ For hiking on the trail to be safe it is necesary but not sufficient that berries are not ripe along the trail and for grizzly bears not to be seen in the area.

$q \to (\neg r \wedge \neg p)$

?

q	r	p	$\neg r \wedge \neg p$	$q \to (\neg r \wedge \neg p)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
F	T	T	T	T
F	F	T	F	T
F	T	F	F	T

F F F T T

(1a) Determine wether each of the conditional statements its true or False

(a) If $1+1=2$, then $2+2=5$

False

(b) If $1+1=3$, then dogs can fly

True

(c) If $1+1=3$, then $2+2=5$

True

(d) If monkeys can fly, then $1+1=3$

True

(25) Write each of these statements in the form "If p, then q" in english

(d) It is necesary to walk eight miles to get to the top of Long's Peak.

p

To get to the top of Long's Peak, then, it is necesary to walk 8mi.

q

$p \to q$

ⓘ Yoo will get afree icecream cone, provided that yoo are among the first 100 costumers tomorrow

$p \downarrow$

If yoo are one of the first 100 costuners tomorrow, then yoo will get free icecream. ← q

$p \to q$ or $q \to p$

(2a) State the converse, contrapositive and inverse of each of the statements.

ⓒ When I stay up late, it is necesary that I sleep until noon. $p \to q$

Converse $q \to p$

I sleep untill noon if I stay up late.

Contrapositive: $\neg q \to \neg p$

if I don't stay up late, then I don't sleep till noon.

Inverse $\neg p \to \neg q$

If I didnt sleep until noon, then I didnt stay up late.

(35) Construct a truth table for each of the compound propositions

(f) $(p \oplus q) \rightarrow (p \oplus \neg q)$

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \rightarrow (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

1.1

(3) Translate the given statement into propositional logic using the propositions provided.

You can graduate only if you have completed the requirements of your major and you dont owe money to the university and you don't have an overdue library book.

m : "You owe money to the university"
r : "yo have completed the requirements for your major"
b : "you have an overdue library book"
g : "you can graduate"

$$g \rightarrow (r \land \neg(m \lor b))$$

⑦ Express the system specifications using the propositions p: "The message is scanned for viruses" and q: "the message was sent from an unknown system" together with logical connectives (including negations

ⓐ "The message is scanned for viruses whenever the message was send from an unknown system

$p \to q$

ⓒ "It is necesary to scan the message for viruses whenever it was sent from an unknown system"

$p \leftrightarrow q$

⑬ What a boolean search would you use to look the web pages about New Jersey? What if you wanted to find web pages about beaches in the isle of New Jersey (In the English Channel)?

- New AND Jersey
- Isle AND New AND Jersey

㉙ 28 - 35 relate to Inhabitants of an island on which there are three kinds of people: Knights who always tell the truth, knaves who always lie, and spies (or normals) who can either lie or tell the truth. You encounter 3 people A, B, and C. You know one of each is a Knite a Mive and a spy. Each of the 3 people knows the the type of person each of other two is. For each of these situations if possible, determine if there is a unique solution and determine who the Knave Knight and spy are. When there is no unique solution list all possible solutions or state there are no solutions.

A Says "I am the knight", B sass "I am the Knave" and C says "B is the Knit"

If C is true, it implies B is lying, but if b is lying it violates the rules of Knights. therefore C is Liying.

C is either the knive of normal

B can't be the kight because this implies the kight is lying. Furthermore, if B was the Knave it wold imply B is not lying breaking the code of Knaves

B is the normal

This confirms C is the Knave

and A is the knigtt.

A: Knight
B: Normal
C: Knave

㉟ A says "I am not the spy",
B says "I am not the spy",
C Says "I am not the spy",

If the Kave states it is not a spy, then that means the Knave is telling the truth contradicting the core rules. No solution.

There is no 41-a

(41) Suppose there are sings on the doors of two rooms. The sign on the first door reads "In this room there is a lady, and in the othe one there is a tiger", and the sign on the other door reads "In one of these rooms there is a lady and in one of them a tiger". Suppose that you know one of this sings is true. Wich one has the tyger and the lady?

Let's assume that they are both in separate rooms.

Room 1

p is " In this room there is a lady and in the otherone a tiger".

Room 2

q is " In one of these rooms there is a lady and in the otherone a tyger".

One is true one is false

if p is true then q must be false.
but if q is false, it implies
also that p is false. contradicting
p.

If p is false, then the contradiction
is avoided. therefore p is false,
this implies q is true.

Lady is in Room 2
Tyger is in Room 1

Section 1.3

⑦ Use De Morgan's negation for each
of the statements

ⓒ Mei walks or takes the bus to
class

Mei does not walk, and does not
take the bus to class

ⓓ Ibrahim is smart and hard
working.

Ibrahim is not smart, nor hard working

⑨ For each of the following compound propositions use the conditional-disjunction equivalence to find an equivalent compound proposition that does not involve conditionals

ⓐ $p \to \neg q$

$$p \to \neg q$$

$$\neg p \lor \neg q$$

$$\neg (p \land q)$$ ← De Morgan Laws

ⓑ $(p \to q) \to r$

$$(p \to q) \to r$$

$$\neg (p \to q) \lor r$$

$$(p \land \neg q) \lor r$$

$$r \lor (p \land \neg q)$$

$$(r \lor p) \land (r \lor \neg q)$$

⑪ Show that each of these conditional statements is a tautology by using truth tables.

ⓓ $(p \wedge q) \to (p \to q)$

$A = p \wedge q \quad B = p \to q$

p	q	A	B	$A \to B$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

⑮ Show that each conditional statement is a tautology from ex 11.

$$\left.\begin{array}{c} (p \wedge q) \to (p \to q) \\ \neg(p \to q) \to \neg(p \wedge q) \\ (p \wedge \neg q) \to (\neg p \vee \neg q) \end{array}\right\} \text{logical equivalences}$$

$$\neg(p \wedge \neg q) \vee (\neg p \vee \neg q)$$

$$(\neg p \vee q) \vee (\neg p \vee \neg q) \quad \text{De Morgan's Law}$$

$$(q \vee \neg q) \vee (\neg p \vee \neg p) \quad \text{commotative laws}$$

$$T \vee \neg p \quad \text{Indiepotent Laws}$$

$$T \quad \text{Domination Laws}$$

㉓ Each of the exercises 20 - 32 asks you to show two compositions are logically equivalent.

Show that $\neg p \leftrightarrow q$ and $p \leftrightarrow \neg q$ are logically equivalent

$$\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$$

Logical equivalences

$$(\neg p \land q) \lor (p \land \neg q) \equiv (p \land \neg q) \lor (\neg p \land q)$$

commutative laws ⇒

$$(p \land \neg q) \lor (\neg p \land q) \equiv (\quad \land \neg q) \lor (\neg p \land q)$$

My answer looks different from the book

For $\neg p \leftrightarrow q$ to be true p and q must be True, False or False an True respectively. For $p \leftrightarrow \neg q$ to be true p and q must be True, False or False true, respectively. This infact is exactly the same case for the ladder to be true. therefore both statements are equivalent.

㉙ Show that $(p \to r) \lor (q \to r)$ and $(p \land q) \to r$ are logically equivalent

$$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$$

Table 8 of logical equivalences Involving biconditional statements shows this particular example.

For $(p \to r) \vee (q \to r)$ to be false p and q must be true and r false. See for $(p \wedge q) \to r$ to be false, it requires $p \wedge q$ to be true, and r to be false. exaclty the Same set up as the previous proposition. Therefore they are equivalent.

③⑤ Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

when all p, q, r are false $p \to q$ is True Then $(p \to q) \to r$ is False but $p \to (q \to r)$ is true.

③⑨ Find the dual of each of these compound propositions

a) $p \wedge \neg q \wedge \neg r$

$p \vee \neg q \vee \neg r$

④② Show that the logical equivalences in table 6, exept for the double negation law come impairs, where each pair contains a compound propositions that are duals of eachother

obiously?

Identity Laws

1 $p \wedge T = p$

then

2 $p \vee F = p$

Domination laws

1 $p \vee T \equiv T$

Then

2 $p \wedge F \equiv F$

Idempotent laws

1 $p \vee p \equiv p$

then

2 $p \wedge p \equiv p$

Commutative laws

1 $p \vee q \equiv q \vee p$

then

$p \wedge q \equiv q \wedge p$

enough ...

1.4

① Let $P(x)$ denote the statement "$x \leq 4$"

what are the truth values?

a) $P(0)$

True $0 \leq 4$

b) $P(4)$

True $4 \leq 4$

c) $P(6)$

False $6 \leq 4$ Not true.

⑦ Translate these statements in to English, where $C(x)$ is "x is a comedian" and $F(x)$ is " x is funny" and the domain consists of all people.

a) $\forall x\,(C(x) \rightarrow F(x))$

if everyone is a comedian, then everyone is funny.

ⓓ $\exists x\,(C(x) \wedge F(x))$

Some people are comedians but some people are just funny.

⑮ Determine the truth value of each of these statements if the domain consists of all integers.

a) $\forall n\,(n^2 \geq 0)$

True

b) $\exists n\ (n^2 = 2)$

True $\qquad n = \pm 2$

c) $\forall n\ (n^2 \geq n)$

True

d) $\exists\ n\ (n^2 < 0)$

False

㉓ Translate in two ways each of the statements into logical expression using predicates, quantifiers and logical connectives First let the domain consist of the

↗

x students in your class and second let it consist of all people.

↑

y

b) Everyone in your class is friendly.

$\forall x\ P(x)$

$\exists y\ P(y)$

c) There is one person in your class who was not born in california

$\exists x\ P(x)$

$\exists y\ P(y)$

㉟ Express the negation of each of these statements in terms of quantifiers whiout using the negation symbol

b) $\forall x\ (x \leq 2)$

$\exists x\,(x > 1)$ assuming $x \in \mathbb{Z}$

d) $\exists x\,(x < 0)$

$\forall x\,(x > 1)$ assuming $x \in \mathbb{Z}$

f) $\exists x\,((x < 4) \vee (x > 7))$

True for x in $(-\infty, 4) \cup (7, \infty)$

then

$\forall x\,(4 \leq x \leq 7)$

㊲ Find a counter example, if possible, to universally quantified statements, where the domain of all variables consists of all integers.

a) $\forall x\,(x^2 \geq x)$

$\forall x\,(x \leq x^2)$ ☺

㊺ Determine whether $\forall x\,(P(x) \rightarrow Q(x))$ and $\forall x P(x) \rightarrow \forall x Q(x)$ are logically equivalent.

Assignment 2

- 1.5

① Translate these statements in to English, wheres the domain for each variables consists of all real nombers.

a) $\forall x \exists y (x > y)$

For all numbers, there exist at least one nomber that is smaller.

There's always a smaller number

b) $\forall x \forall y (((x \geq 0) \land (y \geq 0)) \rightarrow (xy \geq 0))$

For all numbers x, y, if x and y are greater or equal to zero then their product is also greater or equal to zero.

The product of positive numbers including zero is positive or zero

c) $\forall x \forall y \exists z (xy = z)$

For all real numbers x, y, there exist at least a number z such that the product of x and y is equal to z

The product of any numbers will always yield another number.

⑦ Let $T(x, y)$ mean that student x likes cousine y, where the domain x consists of students at your school and the domain of y consists of all the cuisines.

a) $\neg T$(Abdallah Hussein, Japanese)

Abdallah Hussein likes Japanese cusine.

b) $\exists x\, T(x, \text{Korean}) \land \forall x\, T(x, \text{Mexican}$

There is at least one student who likes Korean coisine but all like Mexican cuisine.

c) $\exists y\, (T(\text{Monique Arsenault}, y) \lor T(\text{Jay Jonson}, y)$

There is at least one cuisine that Monique A. or Jay J. like

d) $\forall x \forall z \exists y\, ((x \neq z) \rightarrow \neg(T(x, y) \land T(z, y)))$

For all students x and z, there is a cuisine y for, when x and z are different studentnt then it is not the case that student x likes y cuisine and z student likes y cuisines

all students x, z, if they're different people, they don't like the same cuisine.

e) $\exists x \exists z \forall y (T(x,y) \leftrightarrow T(z,y))$

There is at least two students x and z that if and only if x likes y cuisine then z also likes y cuisine.

f) $\forall x \forall z \exists y (T(x,y) \leftrightarrow T(z,y))$

There exist at least a cuisine y such that if and only if x likes/Dislik y cuisine then z also likes/Dislikes y cuisine.

⑮ Use quantifiers and predicates with more than one variable to express these statements.

a) Every computer science student needs a course in descrete mathematics.

$x \in$ All students

$y \in$ Discrete math class

$P(x)$: is a computer scientist

$Q(x,y)$: x needs a y

$\forall x \exists y (P(x) \land Q(x, y))$

d) There is a student in this class who has taken at least a course in computer science.

$x \in$ All students in the class

$y \in$ All courses in computer science

$P(x, y)$: x has taken y

$\exists x \exists y P(x, y)$

㉓ Express each of these mathematical statements using predicates, quantifiers, logical connectives and mathematical operators

a) The product of two negative numbers is a positive

$x, y \in \mathbb{R}$

$\forall x \forall y ((x<0) \land (y<0) \rightarrow (xy<0))$

b) The difference of a real number and itself is zero

$x, y \in \mathbb{R}$

$\forall x \forall y ((x = y) \rightarrow (x - y = 0))$

c) Every positive real nomber has at least two square rooths.

$a, b, x \in \mathbb{R}$

$\forall x \exists! a \exists! b (((x \geq 0) \wedge (a = -b)) \rightarrow ((\sqrt{x} = a) \wedge (\sqrt{x} = b)))$

d) A negative real number does not have a square rooth that is a real number

$x, b \in \mathbb{R}$

$\forall x \neg \exists b ((x < 0) \rightarrow (\sqrt{x} = b))$

㉗ Determine the truth value of each of these statements if the domain for all variables consists of all integers

a) $\forall n \exists m (n^2 < m)$

there is always a larger number

- True

e) $\exists n \exists m (n^2 + m^2 = 5)$

There is a pair of numbers whose sum of squares is exactly equal to 5

e.g. $1^2 + 2^2 = 5$ or $0 + \sqrt{5}^2 = 5$

- True

f) $\exists n \exists m (n^2 + m^2 = 6)$

$0^2 + \sqrt{6} = 6$

- True

h) $\exists n \exists m (n + m = 4 \wedge n - m = 2)$

$$\begin{aligned} n + m &= 4 \\ n - m &= 2 \end{aligned} \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 4 \\ 1 & -1 & 2 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$$

$n = 3 \quad m = 1$

- True

- 1.6

⑦ What rules are used in this famous argument? "All man are mortal, Socrates is a man, Therefor Socrates is mortal".

$x \in$ all man

$P(x)$: x is mortal

$$\frac{\forall x\, P(x)}{\therefore P(c)}$$

Universal Instantiation

⑬ For each arguments express wich rules of inference are used in each step.

c) "Each of the 93 students in this class own a personal computer. Every one who owns a personal computer can use a word processing program. Therefor zeke, a student in this class can use a processing program

$x \in$ people

P: x owns a PC

Q: x can use a word processing program

R: x is a student in the class

$\forall x\, (R(x) \to P(x))$ Premese

$\forall x\, (P(x) \to Q(x))$ Premese

$\forall x (R(x) \to Q(x))$ Hypotetical sylogism

$R(zeke) \to Q(zeke)$ Conclution

⑲ Determine whethere each of these arguments is valid. If an argument is correct, what rule of inference is being used? If is not, what logical errors occurs?

a) If $n \in \mathbb{R}$ such that $n > 1$ then $n^2 > 1$ suppose that $n^2 > 1$ then $n > 1$

Modus ponens

b) If $n \subset \mathbb{R}$ with $n > 3$ then $n^2 > 9$

suppose that $n^2 \leq 9$ then $n \leq 3$

$p: n > 3$

$q: n^2 > 9$

$$\begin{array}{l} \neg q \\ \underline{p \to q} \\ \therefore \neg p \end{array}$$

Modus Tollens.

¬False bc the squares of negative numbers smaller than -3 are larger than 9. The conclusion is False

c) If $n \subset \mathbb{R}$ with $n > 2$ then $n^2 > 2$

Suppose $n \leq 2$ then $n^2 \leq 4$

$p: n > 2$

$q: n^2 > 2$

$$\begin{array}{l} p \to q \\ \underline{\neg p} \\ \therefore \neg q \end{array}$$

Modus tollens

False

Deniying p does not deny q
and False for numbers smaler then -2

㉓ Identify the errors in this argument that supposedly shows that if $\exists x P(x) \land \exists Q(x)$ is true then $\exists x(P(x) \land Q(x))$ is true

1 $\exists x P(x) \lor \exists x Q(x)$ premise
2 $\exists x P(x)$ Simplification (1)
3 $P(c)$ Existential instantiation form
4 $\exists x Q(x)$ Simplification form (1)
5 $Q(c)$ Simplification form (2)
6 $P(c) \land Q(c)$ Conjunction from (3) and (5)
7 $\exists x(P(x) \land Q(x))$ Existential generalization

The error comes from step 1
Simplification comes from a:

$$\frac{p \land q}{\therefore p} \quad \text{Not} \quad \frac{p \cup q}{\therefore p}$$

BUT the boook tells me step 5 bc. the c is not the same for $Q(c)$ and $P(c)$

(27) Use rules of inference to show that $\forall x(P(x) \Rightarrow (Q(x) \land S(x)))$ and $\forall x(P(x) \land R(x))$ are true, then $\forall x(R(x) \land S(x))$ is true

1 $\forall x\,(P(x) \land R(x))$

2 $P(c) \land R(c)$

3 $P(c)$

4 $\forall x(P(x) \Rightarrow (Q(x) \land S(x)))$

I Really don't even know what to do here.

- Sec 1.7

③ Show that the square of an even number is an even number using direct proof

$P(x)$: n is even $\qquad \forall n\,(P(x) \Rightarrow Q(x))$

$Q(x)$: n^2 is even

if n is even, there exist a $2k$ such that $2k = n$, say

$$(2k)^2$$
$$= 4k^2$$
$$= 2(2k^2)$$

by the deffinition of an even number $2(2k^2)$ is even $\therefore$ $Q(n)$ is true.

⑦ Use a proof by contradiction to show that the sum of an irrational number plus a rational number is irrational

x rational, y irrational, s sum

$P(x,y)$: sum of $x + y = s$ is rational

$Q(s)$: irrational

$$\neg p \rightarrow q$$

$\neg P(x,y)$: sum of $x + y = s$ is irational

say

$$x = \frac{a}{b} \qquad s = \frac{m}{n}$$

$$x + y = s$$

$$x - s = y$$

$$\frac{a}{b} - \frac{m}{n} = y$$

$$\frac{an - bm}{bn} = y$$

y is not rational, $\therefore \neg P(x, y)$ is False

$P(x, y)$ is true then $Q(s)$ is true.

⑮ Prove that x is an irrational number and $x > 0$, then $\sqrt{x}$ is also irrational

x is irrational but $\sqrt{x}$ is not irrational

Ö what! yes! $\sqrt{x}$ so I can express it as a fraction, yeah!

$$\sqrt{x} = \frac{a}{b}$$

Let's square it! $x = \frac{a^2}{b^2}$ Hold it!

x is irrational! Not possible If $\sqrt{x}$ cannot be rational

proving by contraposition that indeed the root of a positive irrational numbers also irrational

㉓ Let $P(n)$ be the proposition "If a and b are positive real numbers, then $(a+b)^n \geq a^n + b^n$

Prove that $P(1)$ is true. What kind of proof did you use?

$$(a+b)^n \geq a^n + b^n$$

$$a^n + \binom{1}{n} a^{n-1} b^1 + \dots + \binom{n}{1} a^1 b^{n-1} + b^n \geq a^n + b^n$$

$$\binom{1}{n} a^{n-1} b^1 + \dots + \binom{n}{1} a^1 b^{n-1} \geq 0$$

shows the difference between left and right side and why left is greater then Right

$$P(1): (a+b)^1 \geq a^1 + b^1$$

$$a+b \geq a+b$$

True

Direct proof

㉙ If n is a positive integer, then

is odd only if $5n+6$ is odd

$5n+6$ is even if n is even and $n>0$

$5(2k)+6 = 10k+6 = 2(5k+3)$

even by deffinition

then since $\neg q \rightarrow \neg p$ is true it

follows that $p \rightarrow q$ also true

- Sec 1.8

⑤ Prove that if x and y are real numbers,

then $\max(x,y) + \min(x,y) = x+y$

case 1 $x > y$

$$+\left.\begin{array}{l} \max(x,y) = x \\ \min(x,y) = y \end{array}\right\} x+y$$

Case 2 $x < y$

$$+\left.\begin{array}{l} \max(x,y) = y \\ \min(x,y) = x \end{array}\right\} = y+x = x+y$$

assosiative law

Case 3 $x=y$

$$+\left.\begin{array}{l}\max(x,y) = y,\ x \\ \min(x,y) = y,\ x\end{array}\right]\left.\begin{array}{l}= x+x \\ = y+y\end{array}\right\} = x+y$$

Conclude true

⑨ Prove the triangle inequality. $x, y \in \mathbb{R}$

then $|x|+|y| \geq |x+y|$

① $x \leq |x|$ & $y \leq |y|$ also ② $\sqrt{x^2} = |x|$ $\sqrt{y^2} = |y|$

$$x^2 = |x|^2 \text{ and } y^2 = |y|^2$$

$$(|x|+|y|)^2 = |x|^2 + 2|x||y| + |y|^2$$

$$(|x|+|y|)^2 = x^2 + \overset{①}{2|x||y|} + y^2$$

$$(|x|+|y|)^2 \geq x^2 + 2xy + y^2$$

$$(|x|+|y|)^2 \geq (x+y)^2$$

$$|x|+|y| \geq |x+y|$$

⑲ Suppose a and b are odd integers with $b \neq a$ show that there is a unique integer c such that $|a-c| = |b-c|$

Case 1 Plus

$$a - c = b - c$$

$$a = b$$

$a \neq b$

Case 2 minus

$$a - c = -b + c$$

$$a + b = 2c$$

$$\frac{(a+b)}{2} = c$$

There is at least one solution.

③ Prove that there is no positive integer n such that $n^2 + n^3 = 100$

$f(n) = n^3 + n^2$

$n = 0$	0
$n = 1$	2
$n = 2$	12
$n = 3$	36
$n = 4$	80
$n = 5$	150

Asingment III

- Sec 2.1

① List the members of these sets

c) $\{x \mid x$ is a square of an integer and $x < 100\}$

$\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d) $\{x \mid x$ is an integer such that $x^2 = 2\}$

$\{\emptyset\}$

⑨ For each of the following sets, determine whether 2 is an element on the sets

a) $\{x \in \mathbb{R} \mid x$ is an integer greater than $1\}$

$(2 \in \mathbb{Z}) \wedge (2 > 1)$

2 is an element of the set

e) $\{\{2\}, \{2, \{2\}\}\}$

- 2 is not an element of the set because it only contains sets.

⑬ Determine whether these statements are true or false.

a) $x \in \{x\}$ true

b) $\{x\} \subseteq \{x\}$ true

c) $\{x\} \in \{x\}$ False

d) $\{x\} \in \{\{x\}\}$ True

e) $\emptyset \subseteq \{x\}$ true

f) $\emptyset \in \{x\}$ true

(19) Suppose that A, B and C are sets that $A \subseteq B$ and $B \subseteq C$. Show that $A \subseteq C$.

$\forall a \forall b ((a \in A \wedge b \in B) \rightarrow (a \in B \wedge b \in A))$

$\therefore A = B$

$\forall b \forall c ((b \in A \wedge c \in C) \rightarrow (a \in C \wedge c \in C))$

$\therefore A = C$

(29) Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$ Find

a) $A \times B$

$A \times B = \{(a, y), (a, z), (b, y), (b, z),$
$(c, y), (c, z), (d, y), (d, z)\}$

b) $B \times A$

$B \times A = \{(y, a), (y, b), (y, c), (y, d)$
$(z, a), (z, b), (z, c), (z, d)\}$

(41) Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same

$A \times B \times C$ will create 3-tuples of the form (a, b, c) but $(A \times B)$ will create 2-tuples in the form $((a,b), c)$

- Sec 2.2

(5) Prove the complementation law in Table 1 by showing that $\overline{\overline{A}} = A$

$$\overline{\overline{A}} = \{x \in U \mid x \notin \overline{A}\}$$
$$\overline{\overline{A}} = \{x \in U \mid \neg(x \in \overline{A}\}$$
$$\overline{\overline{A}} = \{\neg(x \notin U \mid \neg(x \in \overline{A}\}$$
$$\overline{\overline{A}} = \neg\{x \notin U \mid x \in \overline{A}\}$$
$$\overline{\overline{A}} = \neg\neg\{x \notin U \mid x \notin \overline{A}\}$$ Double Negation
$$\overline{\overline{A}} = \{x \mid x \notin \overline{A}\}$$
$$\overline{\overline{A}} = \{x \mid x \in A\} = A$$

(11) Let A and B be sets. Prove the commutative laws from table 1 by showing that

a) $A \cup B = B \cup A$

b) $A \cap B = B \cap A$

a) $A \cup B = \{x \mid x \in A \vee x \in B\}$

$= \{x \mid (x \in A) \vee (x \in B)\}$

$= \{x \mid (x \in B) \vee (x \in A)\}$ Commutative laws

$= B \cup A$

b) $A \cap B = \{x \mid x \in A \wedge x \in B\}$

$= \{x \mid (x \in A) \wedge (x \in B)\}$

$= \{x \mid (x \in B) \wedge (x \in A)\}$ Commutative laws

$= B \cap A$

(21) Show that if A and B are sets, then

a) $A - B = A \cap \overline{B}$

b) $(A \cap B) \cup (A \cap \overline{B}) = A$

a) $A - B = \{x \mid x \in A \wedge \underbrace{x \notin B}\}$ *

$\overline{B} = \{x \in U \mid x \notin B\} = \{x \mid x \notin B\}$

By deffinition of the complement: is $\overline{B}$, then also by the deffinition of the union, $A \cap B = \{x \mid x \in A \wedge x \in B\}$ it is possible to say $A - B = A \cap \overline{B}$

c) $(A \cap B) \cup (A \cap \overline{B})$

$= \{x \mid ((x \in A) \wedge (x \in B)) \vee ((x \in A) \wedge (x \notin B))\}$

$= \{x \mid (x \in A) \wedge ((x \in B) \vee (x \notin B))\}$ Distribution

$= \{x \mid (x \in A) \wedge ((x \in B) \vee \neg (x \in B))\}$ Negation Law

$= \{x \mid (x \in A) \cancel{\wedge (x \in A)}\}$ simplification

$= \{x \mid x \in A\} = A$

(27) Let $A = \{0, 2, 4, 6, 8, 10\}$,
$B = \{0, 1, 2, 3, 4, 5, 6\}$ and
$C = \{4, 5, 6, 7, 8, 9, 10\}$ Find

a) $A \cap B \cap C$

c) $(A \cup B) \cap C$

a) $A = \{0, 2, 4, 6, 8, 10\}$
$B = \{0, 1, 2, 3, 4, 5, 6\}$
$C = \{4, 5, 6, 7, 8, 9, 10\}$

$A \cap B \cap C = \{4, 6, \}$

c) $(A \cup B) \cap C = C \cap (A \cup B)$
$= (C \cap A) \cup (C \cap B)$

① $C \cap A = \{6, 8, 10\}$

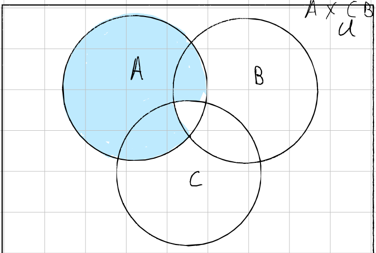
② $C \cap B = \{4, 5, 6\}$

③ $(C \cap A) \cup (C \cap B) = \{5\}$

Book says im wrong, why?

(29) Draw the venn diagrams for each of these combinations

c) $(A \cap \bar{B}) \cup (A \cap \bar{C})$



$A \times (B - C) = (A \times B) - C$

$$(A \cap \overline{B}) \cup (A \cap \overline{C}) = A \cap (\overline{B} \cup \overline{C}) =$$

$$A \cap \neg(B \cap C) = A \cap \overline{(B \cap C)}$$

$$\{(a,b) \mid (a \in A \wedge (b \in B \wedge b \notin C)$$

③⑦ Prove or disprove that for all the sets A, B, C we have

a) $A \times (B - C) \overset{\nearrow}{=} (A \times B) - (A \times C)$

$$A \times (B - C) = (A \times B) - (A \times C)$$

$$(A \times B) - (A \times C) = \{ (m,n) \mid (n \in A) \wedge (m \in B) \wedge (n \in A) \wedge \neg(m \in C) \quad \}$$

$$\{(m,n) \mid (n \in A) \wedge (n \in A) \wedge (m \in B) \wedge \neg(m \in))\}$$

independent laws

$$\{(m,m) \mid (n \in A) \wedge (m \in B) \wedge \neg(m \in C))$$

$$\{(m,n) \mid (n \in A) \wedge ((m \in B) \wedge (m \in \overline{C}))$$

$$= A \times (B - C)$$

⑤⑨ Using the same universal set as in the last exercise, find the set specified by each of these bit strings

a) 11 1 1 00 1111

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$a = 1111\,00\,1111$$

$$a = \{1, 2, 3, 4, 7, 8, 9, 10\}$$

- Sec 2.3

① Why is f not a function from $\mathbb{R}$ to $\mathbb{R}$ if

a) $f(x) = 1/x$

b) $f(x) = \sqrt{x}$

c) $f(x) = \pm\sqrt{(x^2+1)}$

a) $f(x) = 1/x$ $\qquad 0 \in \mathbb{R}$

$f(0) = \text{UND}$

b) $f(x) = \sqrt{x} \quad -1 \in \mathbb{R}^- \in \mathbb{R}$

$f(-1) = i$

$i \notin \mathbb{R}$

c) $f(x) = \pm\sqrt{x^2+1}$

$f(x)$ is not a function

⑤ Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assinged values by the function.

a) The function that assigns to each bit string the number of ones in the string minus the number of zeros in the string.

Domain: All bit strings

Range: $\mathbb{Z}$

b) The function that assigns to each bit string twice the number of zeros in that string

Domain: All bit strings

Range: $\{2n \mid n \in \mathbb{Z}^+ \cup \{0\}\}$

d) The function that assigns to each positive integer the largest perfect square not exceeding this integer

Domain: $\{n \mid n \in \mathbb{Z}^+\}$

Range: $\{m^2 \mid m \in \mathbb{N}\}$

⑨ Find These Values

c) $\lceil -\frac{3}{4} \rceil$

g) $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$

c) $\lceil -\frac{3}{4} \rceil = 0$

d) $\lfloor \frac{1}{2} + \lceil 3/2 \rceil \rfloor$

$\lceil 3/2 \rceil = 2$

$\lfloor \frac{1}{2} + 2 \rfloor = 2$

⑫ Determine whether each of these functions from $\mathbb{Z} \to \mathbb{Z}$ are one to one

a) $f(n) = n - 1$

$f(a) = a - 1$ $\quad a - 1 = b - 1 \quad a, b \in \mathbb{Z}$

$f(b) = b - 1$ $\quad a = b$

$f(a) = f(b)$ Injective

b) $f(n) = n^2 + 1$

$a = 2$

$f(2) = 2^2 + 1 = 5$

$a = -2$

$f(-2) = (-2)^2 + 1 = 5$

$f(2) = f(-2)$

$2 \neq -2$

Not injective

c) $f(n) = n^3$

$a, b \in \mathbb{Z}$

$f(a) = f(b)$

$a^3 = b^3$

$a = b$ injective

d) $f(n) = \lceil n/2 \rceil$

$a = 1$

$f(1) = \lceil 1/2 \rceil = 1$

$b = 2$

$f(2) = \lceil 2/2 \rceil = \lceil 1 \rceil = 1$

$f(1) = f(2)$

$1 \neq 2$

Not injective

⑬ Which functions in exersice 12 are onto

a) $f(n) = n - 1$

$\forall y \exists x (f(x) = y)$

$x - 1 = y$

$x = y + 1$ an integer

surjective

corullary bijective

b) $f(n) = n^2 + 1$

$\neg \forall y \exists x (f(x) = y)$

$\therefore \quad y = -1 \qquad x^2 + 1 = -1$

$x^2 = -2$

Not surjective

c) $f(n) = n^3$

$\neg \forall y \exists x (f(x) = y)$

$\therefore \quad x^3 = y$

$x = \sqrt[3]{y}$

because $\sqrt[3]{y}$ not an integer

For all $y \in \mathbb{Z}$

Not surjective

d) $f(n) = \lceil n/2 \rceil$

$\forall y \exists x (f(x) = y \rceil$

$\therefore \lceil \frac{x}{2} \rceil = y$

$y \in \mathbb{Z}$

surjective

(21) Give an explicit formula from the set of integers to the positive set of integers that is

a) one to one but not onto

$$f(n) = 2^n$$

b) onto but not one to one

$f(n) = |n|$

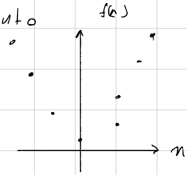
$f(n) | \mathbb{Z} \to \mathbb{Z}^+ \; ; f(a) = b$

not one to one since $b = \{\pm a\}$

but $\forall b \exists a (f(a) = b)$ true

c) One to one and onto

$$f(n) = \begin{cases} 2n & \text{if } n > 0 \\ 1 & \text{if } n = 0 \\ 2|n|+1 & \text{if } n < 0 \end{cases}$$



d) neither one to one, onto

$f(n) = n^2 + 2$ not onto bc.

if $f(a) = 1$

$n^2 + 2 = 1$ $n^2 = -2$

$z_n = \sqrt{-2}$

(29) Show that the function $f(x) = |x|$ from the set of real numbers to the set of non negative real numbers is not invertible, but if the domain is restricted to the set of non negative real numbers the resutant function is invertible.

A function is invertible if and only is bijective.

A function is bijective if and only if is surjective and injective

$f(x) = |x|$ is not injective bc

$$a = 1 \qquad b = -1$$

$$a \neq b$$

$$f(a) = |1| = 1$$

$$f(b) = |-1| = 1$$

$f(x)$ is not invertible

but if $x > 0$ then

$f(x); x>0 = x$

$$f(x) = x$$

$$f(a) = a = f(b) = b$$

$$a = b$$

$f(x); x>0$ is injective

and since $\forall a \forall b \, ((a \in A \wedge b \in B) \rightarrow f(a)=b)$

also $A = B$. $\therefore$ $f(x); x > 0$ is surjective

$\therefore$ $f(x); x>0$ is invertible. □

④① Show that the function $f(x) = ax + b$ from $\mathbb{R} \rightarrow \mathbb{R}$ where a and b are constants with $a \neq 0$ is invertible and find the inverse of f.

- For f to be invertible it must be bijective.

1- $f(x) = ax+b$ is injective

$$f(p) = ap+b = f(q) = aq+b$$

$$ap+b = aq+b$$

$$ap = aq$$

since $a \neq 0$

$$p = q$$

2- $f(x)$ is sorjective

$$\forall y \exists x (f(x) = y)$$

$$ax+b = y$$

$$ax = y-b$$

$$x = \frac{y-b}{a}$$

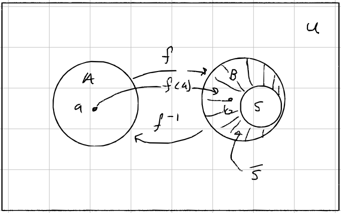
since $a \neq 0$

f is invertible and its inverse is

$$f^{-1}(x) = \frac{x-b}{a} \; ; \; a \neq 0 \qquad \square$$

(47) Let f be a function from the sets A to B. Let S be a subset of B. Show that $f^{-1}(\overline{S}) = \overline{f^{-1}(S)}$

Assuming f is in fact ivertible, we kow that f is bijective



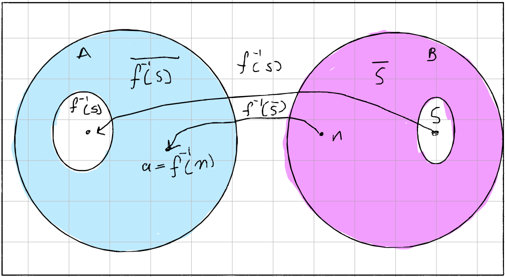
$$\{f^{-1}(\bar{S}) \subset A\} = C$$

$$C = \{\forall b \exists a ((b \notin S \wedge a \in A) \to f(f^{-1}(b)) = a)\}$$

$$\{f^{-1}(S) \subset A\} = D$$

$$D = \{\forall b \exists a ((b \in S \wedge a \in A) \to f(f^{-1}(b)) = a)\}$$

Proof $C = \bar{D}$ $\quad n \in \bar{S}$, $s \in S$



(55) Prove that if n is an integer, then $\lfloor n/2 \rfloor = n/2$ if n is even and $(n-1)/2$ if n is odd

$$2p = n \to p = \frac{n}{2}$$

$$\lfloor 2p/2 \rfloor = \lfloor p \rfloor = p = \frac{n}{2}$$

Since p is an integer

Lemma
- Table 1 Pls a

$\lfloor x \rfloor = n$ if and only if $n \le x < n+1$

if x is odd then $x = 2p+1$

$$n \le 2p+1 < n+1$$

$$n-1 \le 2p < n$$

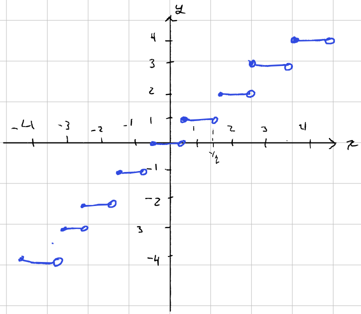
$$\frac{n-1}{2} \le p < n$$

thus if $\lfloor x \rfloor$ is odd then it equals $\frac{n-1}{2}$

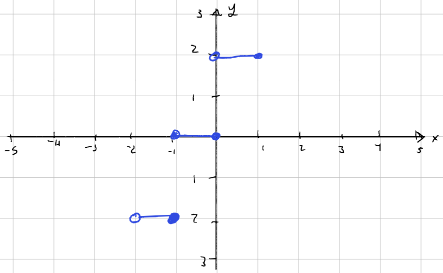
$\square$

(69) Draw the graphs of the functions.

a) $f(x) = \lfloor x + \frac{1}{2} \rfloor$



b) $f(x) = \lceil x - 2 \rceil + \lceil x + 2 \rceil$



Assignment 4

• Section 2.4

⑤ List the first 10 terms of each of the sequences.

d) The sequence whose nth term is $n! - 2^n$

$\{0, -1, -2, -2, 8, 88, 656, 4912, 40064, 362368\}$

f) The sequence whose first term is 2, second term is 4, and each succeeding term is the som of the two preceeding terms.

- The secuence is a Lucas sequence.

$\{2, 4, 6, 10, 16, 26, 42, 68, 110, 178\}$

⑨ Find the five terms of the sequence defined by each of these recoring relations and initial conditions.

c) $a_n = a_{n-1} + 3a_{n-2}$; $a_0 = 1$, $a_1 = 2$

$a_0 = 1$
$a_1 = 2$
$a_2 = 2 + 3(1) = 5$
$a_3 = 5 + 3(2) = 11$
$a_4 = 11 + 3(5) = 26$

$\{1, 2, 5, 11, 26\}$

⑪ Let $a_n = 2^n + 5\cdot 3^n$ for $n \in \mathbb{N} \cap \{0\}$

a) Find a_0, a_1, a_2, a_3, a_4

$a_0 = 2^0 + 5\cdot 3^0 = 6$

$a_1 = 2 + 15 = 17$

$a_2 = 4 + 5\cdot 9 = 4 + 45 = 49$

$a_3 = 8 + 5\cdot 27 = 143$

$a_4 = 16 + 5\cdot 81 = 421$

$\{6, 17, 49, 143, 421\}$

b) Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$
$a_4 = 5a_3 - 6a_2$

$a_2 = 5a_1 - 6a_0 = 5(17) - 6(6) = 85 - 36 = 49$

$a_2 = 49$ ✓

$a_3 = 5a_2 - 6a_1 = 5(49) - 6(17) = 249 - 102 = 143$

$a_3 = 143$ ✓

$a_4 = 5a_3 - 6a_2 = 5(143) - 6(49) =$
$715 - 294 = 421$

$a_4 = 421$

c) Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for
integer's n with $n \geq 2$.

$a_{n-2} = 2^{n-2} + 5\cdot 3^{n-2}$

$a_{n-1} = 2^{n-1} + 5\cdot 3^{n-1}$

$a_n = 5a_{n-1} - 6a_{n-2}$

$a_n = 5(2^{n-1} + 5\cdot 3^{n-1}) - 6(2^{n-2} + 5\cdot 3^{n-2})$

$$a_n = 5(2^{n-1}) + 25(3^{n-1}) - 6(2^{n-2}) - 30(3^{n-2})$$
$$a_n = 5(2^{n-1}) + 25(3^{n-1}) - 3(2^{n-1}) + 10(3^{n-1})$$
$$a_n = 2(2^{n-1}) + 15(3^{n-1})$$
$$a_n = 2^n + 5 \cdot 3^n$$

$$a_n = 5a_{n-1} - 6a_{n-2} = 2^n + 5 \cdot 3^n$$

⑮ Show that the sequence $\{a_n\}$ is a solution to the recurrence relation:

$$a_n = a_{n-1} + 2a_{n-2} + 2n - 9 \quad \text{if}$$

c) $a_n = 3(-1)^n + 2^n - n + 2$

$$a_{n-1} = 3(-1)^{n-1} + 2^{n-1} - (n-1) + 2$$
$$a_{n-1} = 3(-1)^{n-1} + 2^{n-1} - n + 3$$

$$a_{n-2} = 3(-1)^{n-1} + 2^{n-2} - (n-2) + 2$$
$$a_{n-2} = 3(-1)^{n-1} + 2^{n-2} - n + 4$$

$$a_n = a_{n-1} + 2a_{n-2} + 2n - 9$$
$$= (3(-1)^{n-1} + 2^{n-1} - n + 3) + 2(3(-1)^{n-2} + 2^{n-2} - n + 4) + 2n - 9$$
$$= 3(-1)^{n-1} + 2^{n-1} - n + 3 + 6(-1)^{-2} + 2 \cdot 2^{n-2} - \cancel{2n} + \cancel{8} + \cancel{2n} - \overset{-1}{\cancel{9}}$$
$$= -3(-1)^n + 2^{n-1} - n + 2 + 6(-1)^n + 2^{n-1}$$
$$= 3(-1)^n + 2 \cdot 2^{n-1} - n + 2$$
$$= 3(-1)^n + 2^n - 2 + 2$$

$$a_n = a_{n-1} + 2a_{n-2} + 2n - 9 = 3(-1)^n + 2^n - n + 2$$

⑰ Find the solution to each of these recurrance relations and initial conditions. Used the iterative approach such as example 10

d) $a_n = a_{n-1} + 2n + 3 \quad a_{n_0} = 4$

$$a_n = a_{n-1} + 2n + 3$$

$\boxed{a_{n-1} + n = a_n = ?}$

$a_0 = 4$

$a_1 = 4 + 2(1) + 3 = 9$

$a_2 = (4 + 2(1) + 3) + 2(2) + 3$

$= 4 + 2(3) + 3(2) = 16$

$a_3 = (4 + 2(3) + 3(2)) + 2(3) + 3$

$= 4 + 2(6) + 3(3)$

$a_n = a_{n-1} + n$

where ↓

$4 + 2(a_{n-1} + n) + 3n$

$a_4 = (4 + 2(6) + 3(3)) + 2(4) + 3$

$= 4 + 2(10) + 3(4)$

$a_5 = (4 + 2(10) + 3(4)) + 2(5) + 3$

$= 4 + 2(15) + 3(5)$

$4 + 2 + 3 =$

$n^2 + + 4n + 4$ Not correct.

10 ↙

$a_n = 4 + n(n+1)\ 3n - 3 \quad =$

(25) For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that beguines with the given list. Assuming that your formula is correct, determine the next 3 terms in the secuena

b) 1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8 . . .

Start at one, if the number is odd, print once, if even twice

9, 10, 10

f) 3, 5, 8, 12, 17, 23, 30, 38, 47

$a_n = a_{n-1} + n - 1$

2 3 4 5 6 7 8 9 10

3, 5, 8, 12, 17, 23, 30, 38, 47

2 3 4 5 6 7 8 9

I will start n at 2

thus $a_n = a_{n-1} + n$

try $n = 8$
$a_8 = 23 + 8 = 30$ ✓

Next 3 from given sequence are
$a_{11} = 47 + 11 = 58$
$a_{12} = 58 + 12 = 70$
$a_{13} = 83$

$\{58, 70, 83\}$

㉙ what are the values of these sums
d) $\sum_{j=0}^{8} (2^{j+1} - 2^{j})$

$\sum_{j=0}^{8} (2^{j+1} - 2^{j}) = 511$

㉛ what is the value of each of these sums of a geometric progression.
d) $\sum_{j=0}^{8} 2 \cdot (-3)^{j}$

$2\sum_{j=0}^{8} (-3)^{j} = 9842$

㉝ Complete each of these double sums.
b) $\sum_{i=0}^{2} \sum_{j=0}^{3} (2i + 3j)$

$\sum_{i=0}^{2} \sum_{j=0}^{3} (2i + 3j)$

$= \sum_{i=0}^{2} (2i + 2i + 3 + 2i + 6 + 2i + 9)$
$= \sum_{i=0}^{2} (8i + 18) = 78$

- Sec 2.5

① Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one to one correspondance between the set of positive integers and that set.

a) The negative integers

Countable since

$\exists f (f \mid \mathbb{Z}^+ \to \mathbb{Z}^-)$

$f(a) = -a$

$f(b) = -b \quad f(a) = -a$

$-b = -a$

$a = b$ one to one

$\forall a \in \mathbb{Z}^- \exists b \in \mathbb{Z}^+ (f(a) = b)$

$-a = f(a) = b$

$-a = b$

$a = -b = f^{-1}(b)$ onto

$(\forall a \forall b (f(a) = f(b) \to a = b) \land \forall a \in \mathbb{Z}^- \exists b \in \mathbb{Z}^+ (f(a) = b)$

d) the real numbers between 0 and ½

Theorem 1 p 184 states that if A and B are countable then $A \cup B$ is also countable

thus If $A \cup B$ is not countable then it is not the case that, A and B are countable

Suppose $A \cup B = C$ where C is the set of numbers real numbers from 0 to 1. A is the set of real numbers from 0 to ½ and B is the set of real numbers from ½ to 1.

thus, since C is not countable it follows that A and B are also not countable

∴ The set of real numbers from 0 to ½ is not countabl

f) the integers that are multiples of 7 countable since the set has only 1 member

⑪ Give an exumple of two uncountable sets such that $A \cap B$ is

a) Finite

Set $A = \{ a \mid a \in \mathbb{R}^+ \land 0 < a \leq 1/2 \}$

Set $B = \{ b \mid b \in \mathbb{R}^+ \land \frac{1}{2} \leq b < 1 \}$

$A \cap B = \{ 1/2 \}$

b) Countably infinite

Set $A = \{ \{ a \mid a \in \mathbb{R}^+ \land 0 < a < 1/2 \} \cup \mathbb{Z}^+ \}$

Set $B = \{ \{ b \mid b \in \mathbb{R}^+ \land \frac{1}{2} < b < 1 \} \cup \mathbb{Z}^+ \}$

$A \cap B = \mathbb{Z}^+$ $\mathbb{Z}^+$ is coutable.

c) Uncountable

Set $A = \{ a \mid a \in \mathbb{R}^+ \land 0 < a < 1/2 \}$

Set $B = \{ b \mid b \in \mathbb{R}^+ \land 0 < b < 1 \}$

$A \cap B = \{ p \mid p \in \mathbb{R}^+ \land 0 < p < 1/2 \}$

⑰ If A is an uncountable set and B is a countable set, most $A - B$ be uncountable?

Suppose $A = \{a \in \mathbb{R}^+ \wedge 0 < a \leq 1\}$
and and $B = \{1\}$
Then

$$A - B = \{p \mid p \in \mathbb{R}^+ \wedge 0 < p \leq 1 \wedge p \neq 1\}$$
$$A - B = \{p \mid p \in \mathbb{R}^+ \wedge 0 < p < 1\}$$

is uncountable

Book proof is strange

㉓ Show that if A is an infinite set, then it contains a countably infinite subset.

By using the axiom of choise. we select distinct elements from a and make a subset out of them this sub set is then countatble.

• Section 3.1

③ Devise an algorithm that finds the sum of all integers on a list.

```
Procedure: Sum ($\sigma$: Integer, $a_1, a_2\ a_3, \dots, a_n$: integers)
    $\sigma := 0$
    For $i := 1$ to $n$
        $\sigma := \sigma + a_n$

Print ("Sum =" $\sigma$)
```

⑨ A palindrome is a string that reads the same forwards and backwards. Describe an algorithm for determining wheather a string of n characters is a palindrome

```
Procedure Palindrome {$p$: Boolean, $c_1, c_2, \dots, c_n$
: Characters, $i$ ; integer, $j$ : integer)
    $p := $ True
    $i := n$
    $j := 1$
while $p ==$ true and $j \leq \lfloor n/2 \rfloor$
    if $a_j = a_i$ then
        $j := j + 1$
        $i := i - 1$
    else $p :=$ False
return(if $p ==$ true, then
        $c_1 c_2 c_3 \dots c_n$ is a palindrome)
    else
        $c_1 c_2 c_3 \dots c_n$ is not a palindrome)
```

To not double check

Floor Funct bc odd we don't need to check middle term since it would be true if all previous are true, middle term acts like a pivot.

⑬ List all the steps used in search for 9 in sequence 1, 3, 4, 5, 6, 8, 9, 11 using

a) Linear search

$i := 1$

$i := 2$

$i := 3$

$i := 4$

$i := 5$

$i := 6$

$i := 7$

b) Binary search

$i = 1$

$j = 8$

$m = 4$

$i = 5$

$m = 6$

$i = 7$

$m = 7$

Location = i = 7

⑳ Describe an algorithm for finding both the largest and smallest integers in a finite sequence of integers.

```
Procedore Lin-Max-Min (Max, Min, i, $a_1, a_2, \ldots a_n$ : integers);
  max := min := $a_1$
  $i$ := 2
 for $i$ to $n$
    if max < $a_i$ then
          max := $a_i$
    else if min > $a_i$ then
          min := $a_i$
 return "Max=" max , "Min=" min.
```

Section 3.2

① Determine whether each of these functions is $O(x)$

b) $f(x) = 3x + 7$

$|3x + 7| \leq |9 + 1| |x|$

$b = 7 = k$

$C = 3$

$O(x)$

d) $f(x) = 5\log x$

$x^5 < 10^x$

$5\log x < x$

$C = 1$

$k = 1$

$O(x)$

⑦ Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions

a) $f(x) = 2x^3 + x^2 \log x$

$$2x^3 + x^2 \log x \leq 2x^3 + x^2 x$$

$$\log x \leq x$$

say for $x > 1$

$$2x^3 + x^2 \leq 3x^3$$

$C = 3$

$K = 1$

$O(x^3)$ $n = 3$

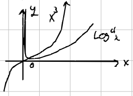
b) $f(x) = 3x^3 + (\log(x))^4$

$$3x^3 + \log^4 x > 3x^3 + x^4$$

$$\log x < x$$

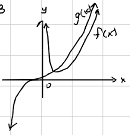
$$\log^4 x < x^3$$

For $x > 1$



$$3x^3 + \log^4 x > 3x^3 + x^3$$

$$3x^3 + \log^4 x > 4x^3$$



$C = 4$

$K = 1$

$O(x^3)$ $n = 3$

c) $f(x) = (x^4 + x^2 + 1)/(x^3 + 1)$

$$\frac{x^4 + x^2 + 1}{x^3 + 1} \leq \frac{2x^4}{x^3} = 2x$$

For $x > 1$

$C = 2$

$K = 1$

$O(x)$ $n = 1$

d) $f(x) = (x^4 + 5\log x)/(x^4 + 1)$

$$\frac{x^4 + 5\log x}{x^4 + 1} < x$$

for $x > 0$

$C = 1$

$K = 0$

$O(x)$ $n = 1$

⑮ Explain what it means for a function to be $O(1)$

that it is constant

Assingment #5

4.1

⑤ Show that if $a | b$ and $b | a$ where a and b are integers then show $a = b$ or $a = -b$

By the fundamental theorem of Arithmetic it necesarilly follows that if $a | b$ and $b | a$ then $\frac{b}{a} = \frac{a}{b} = 1$ meaning

either $b = a$ or $b = -a$

⑪ Prove that if a is an intiger non divisible by 3 then $(a+1)(a+2)$ is divisible by 3

$a = -1$

$(-1+1)(-1+2) = 0$

Thus Not true

⑰ Suppose that a and b are integers and $a \equiv 4 \pmod{13}$ and $b \equiv 9 \pmod{13}$ Find the integer $0 \le c \le 12$ such that

b) $c \equiv 11b \pmod{13}$

$c = 99 \bmod 13$

$c = 8$

d) $c \equiv 2a + 3b \pmod{13}$

$c = 2(4 \bmod 13) + 3(9 \bmod 13 \pmod{13}))$

$c = 8 + 27 \bmod 13$

$c = 8 + 1$

$c = 9$

f) $c \equiv a^3 - b^3 \pmod{13}$

$c = 4^3 - 9^3 \bmod 13$

$c = 16 - 1 = 15$

(29) Find a div m and a mod m when

a) $a = 228 \quad m = 119$

$228 \text{ div } 119 = 1$

$228 \text{ div } 119 = 109$

b) $a = 9009, \quad m = 223$

$9009 \text{ div } 223 = 40$

$9009 \text{ div } 223 = 89$

c) $a = -10101, \quad m = 333$

$-10101 \text{ div } 333 = 30$

$-10101 \bmod 333 = 111$

d) $a = -765432, \quad m = 3871$

$-76432 \text{ div } 3871 = 197$

$-76432 \bmod 3871 = 2845$

㉟ Decide whether each of these integers is congruent to 5 mod 17

a) 80

$8-5=75$ $17 \nmid 75$

Not congruent

b) 103

$103-5=98$ $17 \nmid 98$

Not Congruent

c) -29

$-29-5=-34$ $17 \mid -34$

Congruent

d) -122

$-122-5=-127$ $17 \nmid -127$

Not Congruent

㊴ Find each of these values

a) $(99^2 \bmod 32)^3 \bmod 15 = 9$

b) $(3^4 \bmod 17)^2 \bmod 11 = 4$

c) $(19^3 \bmod 23)^2 \bmod 31 = 25$

d) $(89^3 \bmod 79)^4 \bmod 26 = 0$

㊸ Find counter examples to each of these statements about congruences.

a) If $ac \equiv bc \pmod{m}$ where a, b, c and m are integers with $m \geq 2$, then $a \equiv b \pmod{m}$

what does it mean with a counter example?

Sec 4.2

① Convert the desimal expansion of each of these integers to binary expansion

b) 4532

$4532 \bmod 2 = 0$

$2266 \bmod 2 = 0$

$1133 \bmod 2 = 1$

$566 \bmod 2 = 0$

$283 \bmod 2 = 1$

$141 \bmod 2 = 1$

$70 \bmod 2 = 0$

$35 \bmod 2 = 1$

$17 \bmod 2 = 1$

$8 \bmod 2 = 0$

4 mod 2 = 0

2 mod = 0

1 mod = 1

$$(1\,000\,11\,0\,11\,0\,1\,0\,0)_2 = (4532)_{16}$$

③ Convert the binary expansion of each of these integers to decimal expansion.

c) $(101010101)_2$

$$1\cdot 2^8 + 0\cdot 2^7 + 1\cdot 2^6 + 0\cdot 2^5 + 1\cdot 2^4 + 0\cdot 2^3 + 1\cdot 2^2 + 0\cdot 2^1 + 1\cdot 2^0$$

$$= 2^8 + 2^6 + 2^4 + 2^2 + 2^0 = 256 + 64 + 16 + 4 + 1$$

$$= 341$$

⑪ Convert $(1011\,0111\,1011)_2$ from binary to hexadesimal.

$$(\underbrace{1011}_{B}\ \underbrace{0111}_{7}\ \underbrace{1011}_{B})_2 = (B7B)_{16}$$

⑮ Show that the octogonal expansion of a positive integer can be obtained from its binary expansion by grouping together and translating each block of binary digits in to a single octagonal digit

A random string of 3 bits has a combination total of $2^3 = 8$ thus every possible combination of 3 bits can be expressed as a unique digit in base 8. by making groups of 3 from a binary string and then giving the combinations the corresponding octal digit, any given bit string can be expressed as an octal expansion using zeros to the left as necesary.

Corollary any bit string can be expressed on a base 2^n by making groups of n and following a similar process. where $n \in \mathbb{Z}^+$

⑰ Convert $(734521)_8$ to binary and $(1010111011)_2$ to octal.

$(734521)_8 = (111\ 011\ 100\ 101\ 010\ 001)_8$

7 = 111

3 = 011

4 = 100

5 = 101

2 = 010

1 = 001

$(1\,010\,111\,011)_2 = (1273)_8$

011 = 3

111 = 7

010 = 2

001 = 1

㉑ Find the sum and product of each of these pairs of numbers.

b) $(1110\,1111)_2$, $(1011\,1101)_2$

Sum

$$\begin{array}{cccccccccc}
+ & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
 & & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
\hline
 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0
\end{array}$$

Multiplication

$$\begin{array}{cccccccccccccccc}
 & & & & & & & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
 & & & & & & & \times & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 \\
\hline
 & & & & & & & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
 & & & & & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & \\
 & & & & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & & \\
 & & & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & & & \\
 & & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & & & & \\
 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & & & & & & \\
\hline
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1
\end{array}$$

$$\begin{array}{cccccccccccccccc}
 & & & & & & & & 0 & & 1 & & & & & \\
 & & & & & & 1 & 1 & 1 & 1 & 1 & 0 & 1 & & & \\
 & & & & & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
 & & & & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & \\
 & & & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & & \\
 & & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & & & \\
 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & & & & & \\
 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
\hline
1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1
\end{array}$$

㉓ Find the sum and product of each of these pairs of numbers.

b) $(6001)_8$, $(272)_8$

Sum

$$\begin{array}{r} 6\,0\,0\,1 \\ +\quad 2\,7\,2 \\ \hline 6\,2\,7\,3 \end{array}$$

Multiplication

$$\begin{array}{r} 6\,0\,0\,1 \\ 2\,7\,2 \\ \hline 1\,4\,0\,0\,2 \\ 5\,2\,0\,0\,7 \\ 1\,4\,0\,0\,0\,2 \\ \hline 1\,4\,5\,3\,4\,2\,7\,2 \end{array}$$

(27) Use Algorithm 5 to find 3^{2003} mod 99.

(35) Determine how we can use the desimal expansion of an integer n to determine if an integer is divisible by

a) 2

if the last digit is 0, 2, 4, 6, 8 then it is divisible by two

b) 5 if the last digit is 5 or 0 then it is divisible by 5

c) 10

if the number of digits is more than 3

else if the number's last digit is zero

then it is divisible by 10.

Section 4.3

⑤ Find the prime factorization of 10!

$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10$

$2 \cdot 2^2 \cdot 2^3 \cdot 2 \cdot 2 \; 3 \; 3^2 \; 3 \; 5 \cdot 5 \; 7$

$2^8 \; 3^4 \; 5^2 \; 7$

⑪ Show that $\log_2 3$ is an irrational number.

Suppose $\log_2 3$ is rational then $\exists a, b \in \mathbb{Z}$ such that

$$\log_2 3 = \frac{a}{b}$$

$$b \log_2 3 = a$$

$$\log_2 3^b = a$$

$$3^b = 2^a$$

$$\frac{3^b}{2^a} = 1$$

3^b and 2^a are not factors of eachother for any integers a and b.

$\therefore$ By proof from contradiction, $\log_2 3$ is Not a rational number.

⑰ Determine whether the integers in each of these sets are pairwise relative prime

a) 11, 15, 19

$\gcd(15, 11) = 1$

$\gcd(19, 11) = 1$

$\gcd(19, 15) = 1$

since they are equal, they are relative prime

c) 12, 17, 31, 37

17, 31, 37 are prime, 12 is composite but by the FTA, I conclude these numbers are relative primes

㉕ what are the greatest common divisors of these pairs of integers

a) $3^7 \cdot 5^3 \cdot 7^3, 2^{11} \cdot 3^5 \cdot 5^9$

$GCD = 3^5 \cdot 5^3$

d) $41 \cdot 43 \cdot 53, 41, 43\ 53$

$GCD = 41 \cdot 43 \cdot 53$

f) $1111, 0$

$GCD = 0$

㉗ what is the LCM of each of the pairs from 25.

a) $LCM = 2^{11}\ 3^7 \cdot 5^9 \cdot 7^3$

b) $LCM = 41 \cdot 53 \cdot 41$

f) $LCM = 1111$

㉝ Use the euclidean algorithm to find

b) gdc (100, 101)

$101 = 100 \cdot 1 + 1$ $\quad$ $\gcd(101, 100) = 1$

$101 - 100 = 1$

$100 = 1 \cdot 100$

e) $\gcd(1000, 5040)$

$5040 = 1000 \cdot 5 + 40$

$5040 - 1000 \cdot 5 = 40$ $\quad$ $\gcd(5040, 1000) = 40$

$1000 = 40 \cdot 25$

(39) Using the method followed in example 17, express the greatest common divisor of each of these pairs of integers as a linear combination of these integers.

c) 36, 48

$48 = 36 + 12$

$36 = 12 \cdot 3$

$\gcd(48, 36) = 12 = 48a + 36b \mid a > 0 \quad b = -9$

g) 123, 2347

$$2347 = 123 \cdot 19 + 10$$
$$123 = 10 \cdot 12 + 3$$
$$10 = 3 \cdot 3 + 1$$
$$3 = 1 \cdot 3 + 0$$

$$1 = 10 - (3)3$$
$$1 = 2347 - 123 \cdot 19 - 3(123 - 10 \cdot 12)$$
$$1 = 2347 - 123 \cdot 19 - 123 \cdot 3 + 10 \cdot 12 \cdot 3$$
$$1 = 2347 - 123 \cdot 22 + 10 \cdot 12 \cdot 3$$
$$1 = 2347 - 123 \cdot 22 + (2347 - 123 \cdot 19)36$$
$$1 = 2347 \cdot 37 - 123 \cdot 706$$

㊸ Use the extendet euclidean algorithm to express gdc (26, 91) as a linear combination of 26 and 91

$$91 = 26 \cdot 3 + 13 \qquad \gcd(91, 26) = 13$$
$$26 = 13 \cdot 2 + 0$$

$$13 = 91 - 26 \cdot 3$$

Section 4.4

① show that 15 is an inverse of 7 mod 26

$$7 \cdot 15 = 105$$

$105 = 26(4) + 1$

⑤ Find the inverse of a modulo m for each of the pairs of relatively prime integers using the method followed in example 2

a) $a = 4, m = 9$

$4 \bmod 9$

$\text{gdc}(9,4) = 1$

$9 = 4(2) + 1$

$2 \cdot 4 \bmod 9 = -1$

$-2 \cdot 4 \bmod 9 = 1$

-2 is the multiplicative inverse of $4 \bmod 9$

⑨ Solve the congruence $4x = 5 \bmod 9$ using the inverse of $4 \bmod 9$ found is 5(a)

$4x = 5 \bmod 9$

$-8x = -10 \bmod 9$

$x \equiv 8 \bmod 9$

check

$4x = 32 \bmod 9 \equiv 5 \bmod 9$

⑬ Find the solutions of the congruence $15x^2 + 14x \equiv 5 \pmod{11}$, show that $15x^2 + 14x + 6 \equiv 0 \bmod 11$, factor the LHS show that a solution of the quadratic congruence is a solution of one of the two linear congruences.

$5 \bmod 11 + 6 = 11 \bmod 11 \equiv 0 \bmod 11$

$15x^2 + 19x + 6 = 0 \bmod 11$

$15x^2 + 9x + 10x + 6 = 0 \bmod 11$

$3x(5x+3) + 2(5x+3) = 0 \bmod 11$

$(5x+3)(3x+2) = 0 \bmod 11$

$5x + 3 = 0 \bmod 11$; $3x + 2 = 0 \bmod 11$

$-5x = 3 \bmod 11$

$5x = -3 \bmod 11$

$5x = 8 \bmod 11$

$11 = 5(2) + 1$

$11 - 5(2) = 1$ inverse $= -2$

$x = -16 \bmod 11$

$x = 5 \bmod 11 = 6$

$$3x + 2 = 0 \bmod 11$$
$$3x = -2 \bmod 11$$
$$3x = 9 \bmod 11$$
$$x = 36 \bmod 11$$
$$x = 8 \bmod 11 = 3$$

$$x = \{5 \bmod 11,\ 8 \bmod 11\} \vee \{3, 6\}$$

㉑ use the construction in the proof of the chinise remainder theorem to find all solutions to the system of congruences

$$x \equiv 1 \bmod 2$$
$$x \equiv 2 \bmod 3$$
$$x \equiv 3 \bmod 5$$
$$x \equiv 4 \bmod 11$$

→ 1 $m = 2 \cdot 3 \cdot 5 \cdot 11 = 330$

→ 2 inverses

$165 \bmod 2 \rightarrow y_1 = 1$

$110 \bmod 3$

$2 \bmod 3 \rightarrow y_2 = 2$

$66 \bmod 5 \rightarrow y_3 = 1$

$30 \bmod 11$

$-3 \bmod 11 \rightarrow y_4 = -4$

$x = 1 \cdot 165 \cdot 1 + 2 \cdot 110 \cdot 2 + 3 \cdot 66 \cdot 1 + 4 \cdot 30 - 4 = 323$

$x = 323 \text{ mod } 330$

$x = 323 + 330k \;;\; k \in \mathbb{Z}$

(31) Which integers leave a remainder of 1 when divided by 2 and 1 when divided by 3

$$\begin{cases} 1 = x \text{ mod } 2 \\ 1 = x \text{ mod } 3 \end{cases}$$

$x = 1 \text{ mod } 2$

$x = 1 \text{ mod } 3$

$M = 2 \cdot 3 = 6$

$3 \text{ mod } 2 = 1 \qquad y_1 = 1$

$2 \text{ mod } 3 = -1 \qquad y_2 = -1$

$x = 1 \cdot 3 \cdot 1 + 1 \cdot 2 \cdot -1 = 1 \text{ mod } 6$

$x = 1 + 6k \;;\; k \in \mathbb{Z}$

(39) Use fermat's little theorem to compute 5^{2003} mod 7, 5^{2003} mod 11 and 5^{2003} mod 13

$5^6 \equiv 1 \bmod 7$ 1

$5^{6\cdot 333+5} = \cancel{5^{6\cdot 333}} \cdot 5^5 \equiv 3125 \bmod 7 = 3$

$5^{2003} \bmod 11 \equiv \cancel{5^{10\cdot 200}}^{+3}$ ~~mod~~ $11 \equiv 5^3 \bmod 11 = 4$

$5^{2003} \bmod 13 = \cancel{5^{12\cdot 166}}^{+11} \bmod 13 \equiv 5^{11} \bmod 13 = 8$

㊺ Show that 2047 is a strong pseudoprime to the base 2 by showing it pases Miller's test to the base 2, but is composit

$2^{n-1} = 1 \bmod n$ $\quad 2047 = 23\cdot 89$

$2^{n-1} - 1 = 0 \bmod n$ $\quad 2^d \equiv 2^{1023} \bmod 2047$

$2047 - 1 = 2046 = 2^s \cdot t$

$2^1 \cdot \underbrace{1023}_{t}$

$2^{1023} \bmod 2047$

$2^{102\times 10+3} \bmod 2047$

$(\underbrace{2^{10}}_{1024})^{102} \cdot 8 \bmod 2047$

$\rightarrow [(1024)^2]^{51} \cdot 8 \bmod ..$

$512^5 \cdot 8 \quad \ldots$

8

$(128)^{25} \cdot 2 \bmod \ldots$

㊼ Show that 2821 is a carmichel number

① $2821 = 7 \cdot 13 \cdot 31$

$b^6 \equiv 1 \bmod 7$

$b^{12} \equiv 1 \bmod 13$

$b^{30} \equiv 1 \bmod 31$

$\downarrow$

$$\left(b^{8}\right)^{403} \equiv 1 \mod 7$$
$$\left(b^{12}\right)^{217} \equiv 1 \mod 13 \quad \rightarrow \quad b^{2820} \equiv 1 \mod 2821$$
$$\left(b^{30}\right)^{91} \equiv 1 \mod 31$$

- Section 4.5

③ A parking lot has 31 visitor spaces, numbered 0 to 30. Visitors are assigned parking spaces using a hashing function $h(k) = k \bmod 31$ where k is the number formed by the visitors first 3 digits of their licence plate

a) Which spaces are assigned by the hashing function to cars that have these 3 digits on their license plates:

317, 918, 007, 100, 111, 310?

$h(317) = 317 \bmod 31 = 7$

$h(918) = 918 \bmod 31 = 19$

$h(007) = 007 \bmod 31 = 7$

$h(100) = 100 \bmod 31 = 7$

$h(111) = 111 \bmod 31 = 18$

$h(310) = 310 \bmod 31 = 0$

317 → 8

$918 \rightarrow 20$
$007 \rightarrow 7$
$111 \rightarrow 9$
$111 \rightarrow 9$
$310 \rightarrow 1$

⑪ Find the sewence of pseudorandom numbers generated by the power generator with $p = 11$ $d_1 = 2$ and seed $x_0 = 3$

$$x_{n+1} = x_n^d \bmod p$$

$$x_1 = 3^2 \bmod 11 = 9$$
$$x_2 = 9^2 \bmod 11 = 4$$
$$x_3 = 4^2 \bmod 11 = 5$$
$$x_4 = 5^2 \bmod 11 = 3$$

⑬ Suppose you recived these bit strings over communications link, where the last bit is a parity check bit. In which string are you sure there is an error?

a) $\underbrace{0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1}\ 1$

$$0+0+0+0+0+1+1+1+1+1 \bmod 2 = 1$$
$$5 \bmod 2 = 1$$

- Correct

c) 1 1 1 1 1 1 0 0 0 0 0

$$1+1+1+1+1+1+0+0+0+0 \bmod 2 = 0$$

$$6 \bmod 2 = 0$$

- Correct

⑰ Determine whether the check digit of the ISBN-10 for this textbook, was computed correctly by this publisher.

9 781259676512 ISBN-13

㉙ Determine wheather each of these 15-digit numbers is a valid airline ticket id number

b) 007862342770445

Assingment 6

• Section 5.1

⑤ Prove that $1^2+3^2+5^2+\dots+(2n+1)^2 =$
$\frac{(n+1)(2n+1)(2n+3)}{3}$ for $n > 0$

- $P(n)$ for all non negative n
- Basis step

$$P(0) = 1 = \frac{(0+1)(0+1)(0+3)}{3} = 1$$

- Inductive step

Assume $P(k)$ for an arbitrary k, for $k > 0$

Hipotesis

$$P(k+1) = 1^2+2^2+\dots+(2k+1)^2+(2k+3)^2$$
$$= \frac{(k+2)(2(k+1)+1)(2(k+1)+3)}{3}$$

- Inductive Hipotesis

$$\frac{1}{3}(2k^2+3k+1)(2k+3) + 4k^2+12k+9$$
$$= \frac{1}{3}(4k^3+6k^2+6k^2+9k+2k+3+12k^2+36k+27)$$
$$= \frac{1}{3}(4k^3+24k^2+47k+30) = \frac{1}{3}(k+2)(2k+3)(2k+5)$$

$$= \frac{(k+2)((2k+1)+1)((2k+1)+3)}{3}$$

→ This completes the inductive step

• By mathematical induction, $P(n)$ is true for all integers n with $n > 0$. □

⑨ Find the formula for the first n positive n integers

$\sum n = \frac{n(n+1)}{2}$ → Gauss.

↳ $\sum 2n = 2\sum n = 2\left[\frac{n(n+1)}{2}\right] = n(n+1)$

↑ even

$$\boxed{\sum 2n = n(n+1)}$$

Proof

→ For all positive integers n

→ Basis Step

$P(1) = 2 = (1)((1)+1) = 2$

→ Inductive Step

Assume that $P(k)$ is true for some arbitrary fixed integer $k > 1$

Then the hipotesis says

$P(k+1) = 1+2+\cdots+2k+2(k+1)$

$= (k+1)(k+2)$

Inductive Hipotesis

$= k(k+1) + 2(k+1) = k^2+k+2k+1$

$= k^2+3k+2 = (k+1)(k+2)$

→ This completes the inductive step

• By mathematical induction, $P(n)$ is true for all integers n with $n \geq 1$. □

⑮ Prove for every positive integer n.

$1\cdot 2 + 2\cdot 3 + \cdots + n(n+1) = [n(n+1)(n+2)]\frac{1}{3}$

$P(n)$ For all nonegative integers n

→ Basis step

$P(1) = 1\cdot 2 = \frac{1(2)(\cancel{3})}{\cancel{3}} = 2$ ✓

→ Inductive Step

Assume that $P(K)$ is true for some arbitrary fixed integer $K > 1$

Then Hipotesis

$P(K+1) = 1\cdot 2 + 2\cdot 3 + \cdots + K(K+1) + (K+1)(K+2)$

$= \frac{1}{3}[(K+1)(K+2)(K+3)]$

Inductive Hipotesis

$\frac{1}{3}[K(K+1)(K+2)] + (K+1)(K+2)$

$= \frac{1}{3}[K^2 + 6K^2 + 11K + 6] = \frac{1}{3}[(K+1)(K+2)(K+3)]$

→ This completes the inductive step

- By mathematical induction, $P(n)$ is true for all integers n with $n \geq 1$.

□

㉑ Prove than $2^n > n^2$ for $n > 4$

- For all $n > 4$, $P(n)$

→ Basis Step

$P(5)$ $\quad 2^5 > 5^2$

$32 > 25$ ✓

→ Inductive Step

Assume that $P(K)$ is true for some arbitrary fixed integer $K > 5$

$P(K)$ $\quad 2^K > K^2$

$P(K+1)$ $\quad 2^{K+1} > (K+1)^2$

Inductive Hipotesis

$$2 \cdot 2^K > K^2 \cdot 2$$

$$2^{K+1} > 2K^2 = K^2 + K^2$$

$$K^2 < 2K + 1$$

$$2^{K+1} > K^2 + 2K + 1$$

$$2^{K+1} > (K+1)^2$$

→ This completes the inductive step

- By mathematical induction, $P(n)$ is true for all integers n with $n > 4$.

□

③③ Prove that 5 divides $n^5 - n$ whenever n is a positive integer.

- P(n), For all ositive in ers n

→ Basis Step

$P(0)$ $\quad 0^5 - 0 = 0$ ✓

1
2 2
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

→ Inductive Step

$P(k)$ $\quad k^5 - k$

$P(k+1)$ $\quad (k+1)^5 - (k+1)$

$= (k^5 + 5k^4 + 10k^3 + 10k^2 + 5k^1 + \cancel{1}) - k - \cancel{1}$

$= (k^5 - k) + (5k^4 + 10k^3 + 10k^2 + 5k)$

$= (k^5 - k) + 5(k^4 + 2k^3 + 2k^2 + k)$

Divisible by 5 by previous induction

Dibisible by 5.

• Section 5.2

③ Let P(n) be the statement that a postage of n cents can be formed using just 3 cent stamps and 5cent stamps. The parts of this exercise outline a strong induction proof that P(n) is true for integers $n \geq 8$.

a) Show that the statement $P(8)$, $P(9)$ and $P(10)$ are true completing the bisis step of a proof by strong induction that $P(n)$ is true for all integers $n \geq 8$.

$P(8) \rightarrow 5¢ + 3¢ = 8¢$

$P(9) \rightarrow 3 \cdot 3¢ = 9¢$

$P(10) \rightarrow 2 \cdot 5¢ = 10¢$

b) What is the inductive hypotesis of a proof by strong induction that $P(n)$ is true for all integers $n \geq 8$?

The inductive hypotesis is the statement that $P(j)$ is true: j cents can be forwred using 3 and 5 cent stamps for $8 \leq j \leq k$

c) What do you need to prove in this inductive step of a proof by strong induction that $P(n)$ is true for all integers $n \geq 8$?

That it is possible to form a $P(k+1)$ stamp only using 3¢ and 5¢ stamps.

$xy - x - y +$

d) complete the inductive step for $k \geq 10$

Assume $P(K)$ is true, Then $P(K-2)$ is also true, then

$$P(K+1) = P(K-2) + 3¢$$

e) Explain why these steps show that $P(n)$ is true whenever $n \geq 8$

By completen the steps required bo the principle of strong induction, I have shown it to be true

⑤ Determine which ammounts of postage can be formed by using just 4 cent and 11 cent stams.

4, 8, 11, 12, 15, 16, 19, 20, 22, 23, 24, 26, 27, 28, 30

You can form any value over $n \geq 30$

b) Prove your answer to (a) using the principle of mathematical induction

It is possible to obtain any integer whith enough stamps as long as that given integer is ≤ 30

→ Basis Step

$P(30) = 2 \cdot 11 + 2 \cdot 4 = 30$¢

→ Inductive step

Case 1

$P(K) = 3 \cdot 11 + R$

where R is some remainder

Then $P(K+1) = 3 \cdot 11 + R + 1$

$3 \cdot 11 + 1 = 34 = 4 \cdot 2 + 11 \cdot 2$

$P(K+1) = 11 \cdot 2 + 4 \cdot 2 + R$

Case 2

$P(K) = 4 \cdot 8 + R$

$P(K+1) = 4 \cdot 8 + 1 + R$

$P(K+1) = 11 \cdot 3 + R$

This completes the inductive step. By mathematical induction $P(n)$ is true for integers $n \geq 30$.

Strog induction step

It is possible to obtain any integer whith enough stamps as long as that given integer is ≤ 30

→ Basis Step

$P(30) = 2 \cdot 11\text{¢} + 2 \cdot 4\text{¢} = 30\text{¢}$
$P(31) = 11\text{¢} + 5 \cdot 4\text{¢} = 31\text{¢}$
$P(32) = 8 \cdot 4\text{¢} = 32\text{¢}$
$P(33) = 3 \cdot 11\text{¢} = 33\text{¢}$
$P(34) = 2 \cdot 11\text{¢} + 3 \cdot 4\text{¢} = 34\text{¢}$
$P(35) = 11\text{¢} + 6 \cdot 4\text{¢} = 35\text{¢}$

→ Inductive step

Assume that $P(j)$ is true for $30 \leq j \leq k$, Then it is also the case that $P(k-3)$ is true.

Since $P(k+1) = P(k-3) + 4\text{¢}$, it rollows that $P(k+1)$ is also true.

⑦ Which ammounts of money can be formed usind two dollar bills and 5 dolar bills

$$xy - x - y + 1 = \text{min}$$

$$2 \circ 5 - 2 - 5 + 1 = 10 - 7 + 1 = 4$$

It is possible to make a payment or any denomination as long as said payment is greater than 4

→ Basis Step

$P(4) = 2 \circ 2\,\$ = 4\,\$$

$P(5) = 5\,\$$

$P(6) = 3 \circ 2\,\$ = 6\,\$$

$P(7) = 5\,\$ + 2\,\$ = 7\,\$$

→ Inductive Step.

Assume that it is possible to make any denomination j up to k $4 \leq j \leq k$ then $P(k-1)$ would be true.

Since $P(k+) = P(k-1) + 2\,\$$ It is possible to claim by the principle of strong induction that $P(k+1)$ is true

㉕ Suppose $P(n)$ is a propositional function. Determine for which positive integers n the statement $P(n)$ must be true, and justify your answer.

a) $P(1)$ is true; For all positive integers $P(n)$, if $P(n)$ is true, then $P(n+2)$ is true.

True by the principle of induction.
- However note that It is not possible to conclude the validity of even n's since true statements are constructed in the form $P(\underbrace{1+2})$ $P(\underbrace{3+2}) \rightarrow P(5+2) \cdots P(2K+1) \ni K \in \mathbb{Z}^+$

b) $P(1)$ and $P(2)$ are true; For all positive integers n, then if $P(n)$ and $P(n+1)$ are true, then $P(n+2)$ is true.

-True, since $P(j)$ is true then $P(j+1) = P(K)$ then $P(K+1) = P(j+2)$ that is also true.

c) $P(1)$ is true; for all positive integers n, if $P(n)$ is true, then $P(2n)$ is true.

- Only true if $n = 2, 4, 8 \cdots 2^k$ $k \in \mathbb{Z}$ but the validity of $P(n)$ for $n \notin J$ in conclusive.

d) $P(1)$ is true for all integers positive integers n, if $P(n)$ is true, then $P(n+1)$ is true.

- True, since this is the definition of mathematical induction

• Section 5.3

① Find $f(1)$, $f(2)$, $f(3)$ and $f(4)$ if $f(n)$ is defined recursively by $f(0) = 1$ and $n = 0, 1, 2, \ldots$

c) $f(n+1) = 2^{f(n)}$

$f(1) = 2$

$f(2) = 2^2 = 4$

$f(3) = 2^{2^2} = 16$

$f(4) = 2^{2^{2^2}} = 65536$

d) $f(n+1) = f(n)^2 + f(n) + 1$

$f(1) = 1$

$f(2) = 1^2 + 1 + 1 = 3$

$f(3) = 3^2 + 3 + 1 = 13$

$f(4) = 13^2 + 13 + 1 = 183$

③ Find $f(2)$, $f(3)$, $f(4)$ and $f(5)$ if $f(n)$ is defined recorsively by $f(0) = -1$, $f(1) = 2$ and $n = 1, 2, \ldots$

b) $f(n++1) = f(n)^2 f(n-1)$

$f(0) = -1$

$f(1) = 2$

$f(2) = 2^2(-1) = -4$

$f(3) = (-4)^2(2) = 16$

$f(4) = (16)^2(-4) = -1024$

$f(5) = (-1024)^2(16) = 16777216$

⑤ Determine whethere each of the proposed definitions is a valid recorsive definition of a function f from the set of non negative integers to the set of integers. If f is well defined, find a formula for $f(n)$ when n is a nonegative integer and prove that your formula is valid.

a) $f(0) = 0$, $f(n) = 2f(n-2)$ for $n \geq 1$

$f(2) = 2\boxed{f(2-2)} = 0$

$f(4) = 2f(2) = 0$

Not a valid recursive deft from

$f(n) \mid n \in \mathbb{Z}^+ \cup \{0\} \rightarrow f(n) \in \mathbb{N}$

c) $f(0) = 2$, $f(1) = 3$ $f(n) = f(n-1) - 1$

$n \geq 1$

$f(2) = 3 - 1 = 2$

$f(3) = 2 - 1 = 1$

$f(4) = 1 - 1 = 0$

$f(5) = 0 - 1$

$f(n) = 4 - n$ for $n \geq 2$

$f(n-1) = 4 - (n-1) = 5 - n$

Recursive Formula

$f(n) = (5 - n) - 1 = 4 - n = f(n)$

valid for $n \geq 2$

e) $f(0) = 1$, $f(n) = 3f(n-1)$ if n is odd and $n \geq 1$ and $f(n) = 9f(n-2)$ if n is even and $n \geq 2$

valid

Basis Step

$f(1) = 3$

$f(2) = 9 \cdot (1) = 9$

$f(3) = 3 \cdot 3 = 9$

$f(4) = 9 \cdot 9 = 81$

Inductive Step

There is a true value for any given n. Assume $f(K)$ is true given that $0 \leq j \leq K$. Then $f(K-2)$ is true.

then

$f(K+1) = 3 \cdot f(K)$ for even K

$f(K+1)$ is true

$f(K+1) = 9 f(K-1)$ for odd K

$f(K+1)$ is true

⑪ Give a recursive definition of $P_m(n)$ the product of the integer m and the nonegative integer n.

$P_m(n) = P(n-1) + P(n)$

$P_m(n) = P_m(n-1) + m$

given that $P_m(1) = m$

⑬ Where f_n is the nth Fibonacci number:

Prove that $f_1 + f_3 + \ldots + f_{2n-1} = f_{2n}$ when n is a positive integer.

The fibonacci Sequence is defined by

$f_n = f_{n-1} + f_{n-2}$

$$f_{2n} = f_{2n-1} + f_{2n-2}$$

$$\downarrow$$

$$f_{2n-2} = f_{2n-3} + f_{2n-4}$$

$$\downarrow$$

$$f_{2n-4} = f_{2n-5} + \cdots$$

$$\downarrow$$

$$f_{2n} = f_{2n-1} + \cdots + f_3 + f_1$$

Note how even n terms will always yield another even term, and a left over odd term. Continuing this progression untill reaching 1

(25) Give a recursive deffinition for

a) The set even of integers

$0 \in S$ and if $x \in S$ then $x+2 \in S$ and $x-2 \in S$.

b) The set of positive integers congruent to 2 modulo 3

$(-1 \in S \wedge x \in S) \to (x \pm 3 \in S)$

c) The set of positive integers not divisible by 5

$(1 \in S \wedge 2 \in S \wedge 3 \in S \wedge 4 \in S \wedge x \in S)$
$\to (x+5 \in S)$

27 Let S be the set of positive integers defined by basis step: $5 \in S$
Recursive Step: If $n \in S$, then $3n \in S$ and $n^2 \in S$.

a) show that if $n \in S$, then $n \equiv 5 \bmod 10$

$3 \cdot 5 = 15 \to 5 + 2(10)$

$3 \cdot 15 = 45 \to 5 + 4(10)$

$\vdots$

$5 + R(10)$

a number that is a multiple of 5 times 3 will always have the last digit be 5, and this −5 will always be divisible by 10.

b) Show that there exists an integer $m \equiv 5 \bmod 10$ that does not belog to S

$35 = 30 + 5 \bmod 10 \equiv 5 \bmod 10$

35 /

㉛ Give a recursive definition of each of these sets of oreded pairs of positive integers Use structural induction to prove that the rewursive deffrinition you found is correct [Hint: To find a recursive deffinilion, plot the points and see for patterns.]

a) $S = \{(a, b) \mid a \in \mathbb{Z}^+, b \in \mathbb{Z}^+,$ and $a + b$ is even$\}$

$f(0) = 2$ $\quad$ $f(n) = f(n-1) + 2$

$f(1) = 4$

Assingment # 7

- Section 6.1

① There are 18 mathematics majors and an 325 computer science majors at a college.

a) In how many ways can two reprecentatives be picked so that one is a mathematics major and another one is a computer science major?

$$18 \times 325 = 5850$$

In 5850 different ways.

b) In how many ways can one repreecentative be picked who is either a mathematics major or a computer science major?

Assoming that there are no people who are both a mathematics and a computer science majors then: $18 + 325 = 343$

In 343 different ways

③ A multiple choise test contains 10 questions. there are four possible answers in each question

a) In how many ways can a student answer the questions on the test if if the student answers every question

4^{10} ways

b) in howmany ways can the student answer the test if the student can leave answers blank

Now the number of options per question is 5, thus we have 5^{10} ways

⑦ How many 3 letter initials can people have?

I don't understand what a 3 letter initial means, but I interpret the problem as saying that howmany combinatio
s of 3 letters can be make

- Assuming we're using an alphabet with 27 letters and choosing only caps we have 27^3

⑪ How many bit strings of length 10 end and begin with 1

$\underline{1}\ \underline{x_2}\ \underline{x_3}\ \underline{x_4}\ \underline{x_5}\ \underline{x_5}\ \underline{x_6}\ \underline{x_7}\ \underline{x_8}\ \underline{x_9}\ \underline{1}$

there are only 8 free bits thus 2^8

㉓ How many positive integers between 100 and 999 inclusive

a) are divisible by 7

There are 14 numbers divisible by 7 from 0 → 100

There are 142 numbers divisible by 7 from 0 → 999

Thus $-14 + 142 = 128$

There are 128 numbers divisible by seven from 100 → 999

c) have the same decimal digits

there are 9

d) Are not divisible by 4?

0 → 99 divisible by 4 → 24
0 → 999 divisble by 4 → 249

There are 225 divisible by 4 numbers from 100 → 999, then there are 675 numbers Not divisible by 4

e) are divisible by 3 or 4?

Divisible by 4 : 225
Divisible by 3 : 300
Divisible by 12 : 75

$$\begin{array}{r} +\,225 \\ +\,300 \\ -\,75 \\ \hline 450 \end{array}$$

There are 450 numbers divisible by 4 or

㉗ A commetee is formed consisting of one reprecentative from each of the 50 states in the United States, where the reprecentative of a state is either the goveaor or one of two seaators from that state, how many ways are there of making this comeely

3^{50} ways

(29) how many licence plates can be made using either two upporcase English letters followed by four digits of two digits followed by four upper case English letters?

$$\underline{A}\;\underline{A} - \underline{0}\;\underline{0}\;\underline{0}\;\underline{0} = 26^2 \times 10^4$$

$$\underline{0}\;\underline{0} \rightarrow \underline{A}\;\underline{A}\;\underline{A}\;\underline{A} = + \; 10^2 + 26^4$$

$$52457600$$

52'457,600 ways

(41) A palindrome is a string whose reversal are identical to the string. How many bit strings of leinght n are palindroomes?

half of all bit strings leinght n are palindromes, thus there are only $2^{n/2}$ ways odd n are no problem since the middle odd term is a pivot.

(57) Suppose that a password for a computer system must have at least 8 but no more than 12 characters, where each Caracter is an upper case english letter lowercase, six special characters

$26 + 26 + 6 = 58$

$58^8 + 58^9 + 58^{10} + 58^{11} + 58^{12} = 1.47 \times 10^{21}$

(67) How many ways to arrange the letters a, b, c, and d such that a is not followed immediatly by a b

4! total permutations 24
minus 3! total permutations where ab is found
thus $24 - 6 = 18$

Section 6.2

③ A drawer contains a dozen brown socks and a dozen black socks all unmatched, a man takes socks at random in the dark

a) How many socks must he take out to be sure he has two socks of at least the same color?

$\left\lceil \frac{N}{K} \right\rceil = 2$ $\quad K = 2 \quad$ $\frac{x}{2} = 2$

$x - K + 1 = N$ $\quad$ $x = 4$

$4 - 2 + 1 = 3$

$N = 3$

He must take out a minimum of 3 socks to ensure he gets out a pair of the same colour.

b) How many sock must he take out to be sure that he has at least two black socks?

worst case senario we pull out all of the brown socks, then it's left only brown, we pull 2 more to be sure.

Must pull out 14 socks

⑦ Show that Among any group of five (Not necessarily consecutive) integers, there are 2 with the same remainder when divided by 4

$b \bmod 4$ can only be $0, 1, 2, 3$
4 options total, since im given 5 integers I can prove that I will have 2 with the same remainder by Theorem 1 p 420 (Nice)

⑪ What is the minimum number of students each of whom comes from one of 50 states, who must be from a university to guarantee that there are at least 100 who come from the same state.

Asuming that every state has the same number of people living in it then

$\lceil \frac{N}{K} \rceil = 100$ $K = 50$

$x = 5000$

$N = x - K + 1$

$N = 5000 - 50 + 1 = 4951$

I need 4951 students

⑰ How many numbers from the set $\{1, 2, 3, 4, 5, 6\}$ to guarantee that at least one pair of these numbers adds up to 7

6	5	4	3	2	1
1	2	3	4	5	6
2	3	4	5	6	
3	4	5	6		
4	5	6			
5	6				
6					

Note $(6, 1) \neq (1, 6)$ even tho $1 + 6 = 6 + 1$

- 21 out of 36 pairs add up to 7

15 bad ones worst case, +1 good one left

16 draws total

㉑ Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophmore and a Junior.

a) Show that there are at least nine freshmen, at least a sophmore and a Junior.

If there where at least 8 Juniors 8 sophmores and 8 freshmen we would only have 24 people contradicting the given fact that we only got 25 people

㉝ Show that there are at least six people in California (Population 39 million) with the same 3 initials who where born in the same day of the year (but not the same year) asume that everyone has 3 initials

What about people born on a lap year? lap day?

$26^3 \times 365 = 6415240$

$$\frac{39\times10^6}{26^3\times365} = 6.079273729$$

Section 6.3

③ How many permutations of $\{a, b, c, d, e, f, g\}$ end with a?

Fixing a in the last position then permuting 6 elements

$6! = 720$

⑦ Find the number of 5 permutations of a set with 9 elements

$9P5 = 15120$

⑪ How many bitstrings of length 10 contain

a) exactly 4 1's?

$10C4 = 210$

b) at most 4

$(10C4) + (10C3) + (10C2) + (10C1) + (10C0)$

$= 386$

c) At least four 1?

$10C10 + 10C9 + 10C8 + 10C7 + 10C6 + 10C5 + 10C4 =$

$= 848$

d) equal number of zeros to 1's

$10C5 = 252$

⑮ th How many ways can a set of five letters be selected from the english alphabet

$26C5 = 65780$

(1q) A coin is fipped 10 times where each flip comes up to either heads or tails How many posible outcomes

a) are there in total?

2^{10}

b) contain exactly 2 heads

$10C2 = 45$

c) contain at most three tails?

$10C0 + 10C1 + 10C2 + 10C3 = 176$

d) contain the same number of head and tails

$10C5 = 252$

(2q) A club has 25 members

a) How many ways are there to choose 4 members? $25C4 = 12650$

b) How many ways are there to choose a precident secretary, vp, tresurer, no more than 1 office per person

$25P4 = 303600$

(3q) The english alphabet contains 21 constants and 5 vowels, How many strings of of the lower case alphabet contain

a) exactly one bowel

21^{5}

c) at least one bowel

$26^6 - 21^6 = 22314555$

④① How many licence plates consisting of 3 letters followed by 3 digits contain no letter or digit twice?

$26^3 - 26 \cdot 25 \cdot 3 = 15626$

$1000 - 270 = 730$ $\times$

11406980

Assingment 8

• Section 6.4

③ Find the expansion of $(x+y)^5$

$\binom{5}{0}x^5 + \dots + \binom{5}{5}y^5$

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

⑦ what is the coeficient of x^9 in $(2-x)^{14}$?

$\binom{19}{9}2^{10}x^9 = 92378 \times 2^{10}x^9$

$94595072x^9$

⑬ Use the binomial theorem to find the coeficient of $x^a y^b$ in the expression of $(2x^3-4y^2)^7$
where:

d) $a = 12$ $b = 6$

$\Omega x^{12}y^{6}$

$\Omega (x^3)^4(y^2)^3$ $4+3=7$ ✓

$\Omega = \binom{7}{3}(2)^4(-4)^3$

$\Omega = -35840$

For $a=12$ $b=6$ ⇒ $-35840x^{12}y^6$

⑲ show that $\binom{n}{k} \le 2^n$ for all positive integers n and all integers k with $0 \le k \le n$

Basis Step

P(1) say $\binom{n}{0} = 1 \geq 2^n$

true for all $n \geq 0$

Assume that

$$\binom{n}{0} \le 2^n$$

$$\binom{n}{1} \le 2^n$$

$$\vdots$$

$$\binom{n}{k-1} \le 2^n$$

$$\binom{n}{n} \le 2^n$$

is true for $0 \le k \le n$ then

Inductive Step

suppose

$$\binom{n}{k-1} \le 2^n$$

$$\binom{n}{k-1}\binom{n}{k} \le 2^n + 2^n$$

Pascal's identity

$$\binom{n+1}{k} \le 2(2^n) = 2^{n+1}$$

$$\binom{n+1}{k} = 2^{n+1}$$

This shows by strong induction that $\binom{n}{k} \le 2^n$ for any n and any K.

$\square$

(29) Let n be a positive integer show that.

$$\binom{2n}{n+1} + \binom{2n}{n} = \binom{2n+2}{n+1} \Big/ 2$$

$$\binom{2n}{n+1} + \binom{2n}{n} = \binom{2n+1}{n+1} = \frac{1}{2}\left[\binom{2n+1}{n+1} + \binom{2n+1}{n+1}\right]$$

- Section 6.5

③ How many strings of six letters are there?

By the theorem 1 we got 26^6 ways allowing for repetition.

(9) A begel shop has onion bagels, poppyseed bagels, egg bagels, salty bagels, pumpernikle bagels, sesame seed bagels, raisin bagels, and plain. How many wase are there to choose

a) six bagels?

8 types of bagels and 6 slots.

$C(C(8+6-1, 6)$

$CC(14,6) = 3003$

There are 3003 different combinations.

d) A dozen bagel with at least one of each kind

Removing the 1 of each condintion we are left with 6 spots 6 flavors

$$b_1 + b_2 + b_3 + b_4 + b_5 + b_6 = 6$$

$$x_1 + \dots + x_n = a$$

$$C(C(m+a-1, a)$$

is the solution for this kind of problem.

$$C(6+6-1, 6)$$

$$CC(11, 6) = 462$$

462 ways $\square$

⑬ A book publisher hass 300 copies of a discrete mathematics book. How many ways are there to store these books in their 3 wearhouses if the copies are indistigushable?

$$x_1 + x_2 + x_3 = 3000$$

$$C(3000+3-1, 2) = 4504501$$

㉓ How many ways are there to distribute six indistinguishable balls in to nine indistinguishable bins?

$$x_1 + x_2 + x_3 + \dots + x_9 = 6$$

$C(14, 9) = 2002$ ways

㉕ How many ways are there to distribute 12 distinguishable boxes so that two objects are placed in each box?

$$C(12,2)\,C(10,2)\,C(8,2)\,C(6,2)\,C(4,2)\,C(2,2) = 7484400$$

㉝ How many different strings can be made from letters in abracadabra using all letters

ABRACADABRA

$A = 5 \quad B = 2 \quad R = 2 \quad C = 1 \quad D = 1$

Letters total = 11

$c(11,5)\,c(6,2)\,c(4,2)\,c(2,1)\,c(1,1)$

83160 ways

④⑦ How many ways can n books be placed on K distingushable shelves?

a) if the books are indistinguishable copies of the same title?

$x_1 + \circ\circ\circ\, x_K = n$

the solution is well known to be

$c(K+n-1, K) = C(K+n-1, n-1)$

b) If no two books are the same and the position in the shelve matters?

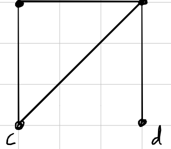
$$\frac{(n+K-1)!}{(K-1)!}.$$

- Section 10.1

③ Determine whether the graph shown has directed or undirected edges, multiple edges, and whether there's a loop. Detemine the type of graph

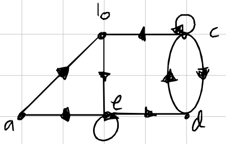
a

b



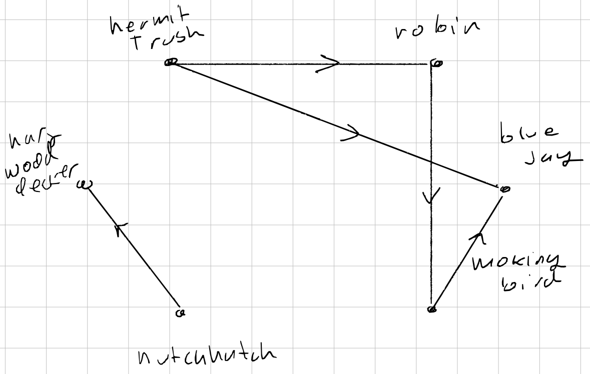
Simple graph

⑦



Mixed Graph

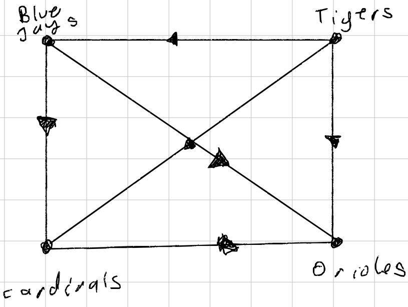
⑮ Construct a niche overlap graph for six species of birds, where the hermit trush competes with the robin and the blue jay, the robin also computes whith the moking bird, the moking bird also competes with the blue jay, and the nutchatch competes with the hairy wood pecker.



㉓ In a round Robin tournament the Tigers beat out the Blue Jays, The Tigers Beat out the cardinals, The Tigers beat out the orioles, the Blue Jays beat the cardinals, the blue Jays beat the orioles and the cardinals beat out the orioles. Modle this outcome with a direct graph

Roster

- Blue Jays
- Tigers
- Cardinals
- Orioles



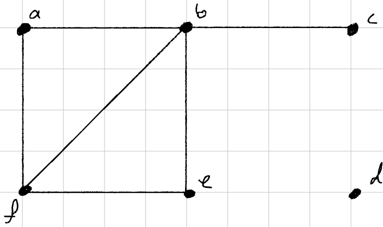
Go Tigers!

(2a) Describe a graph that represents whether each person at a party knows the name of each person at the party. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

To all the questions, it would depend on what the inteded purpose of the graph is and the tipe of interpretation that needs to be achived.

•Section 10.2

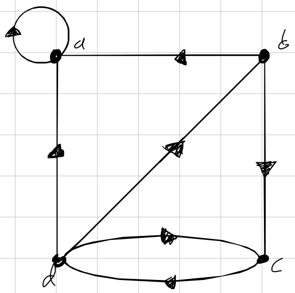
① Find the number of vertices, the number of edges, and the number of each vertex in the given undirected graph. Identify all isolated and pendant vertices



6 vertices
6 edges

$\deg(a) = 2$
$\deg(b) = 3$
$\deg(c) = 1$ → Pendant
$\deg(d) = 0$ → Isolated
$\deg(e) = 2$
$\deg(f) = 3$

⑦ Determine the number of vertices and edges and find the in-degree and out degree of each vertex for the given directed graph.



$|V| = 4$
$|E| = 7$

$\deg^-(a) = 3$
$\deg^+(a) = 1$

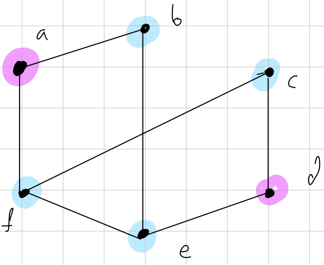
$\deg^-(b) = 1$
$\deg^+(b) = 2$

$\deg^-(c) = 2$
$\deg^+(c) = 1$

$\deg^-(d) = 1$
$\deg^+(d) = 3$

(25) Determine whether the graph is bipartite. You might find useful to apply theorem number 4 and andswer the question by determining whether it is

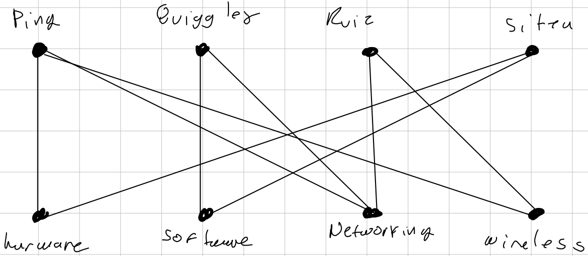
possible to assign either red or blue to each vertex so that no adjecent vertices are assigned the same color.



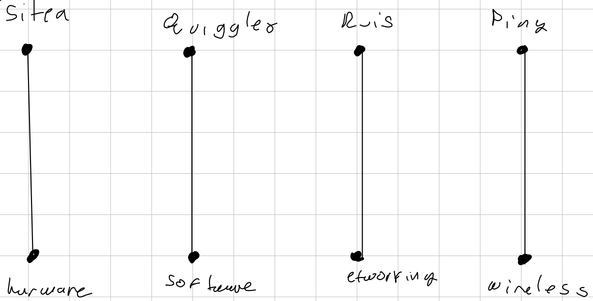
No Bipartite since it is not possible to have theorem 4 before

㉗ Supose that there are four employees in the computer support group of the school of Engineering of a large university. Each employee will be assigned to support one of four different areas hardware, software, networking, and wireless. Suppose that Ping is qualified to support hardware networking and wireless; Quigley is qualified to support networking and wireless and Sitea is qualified to support hardware and software

a) Use a bipartite graph to model the four employees and their qualifications.



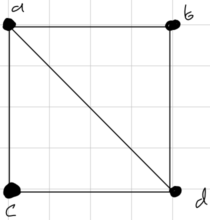
c) If an assignment of employee support areas so that each employee is assigned to one suppor area exists find one



Assignment 9

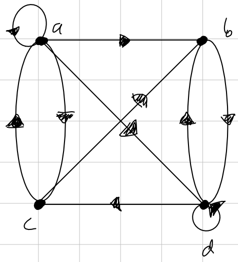
- Section 10.3

① Use an adjacency list to represent the given graph



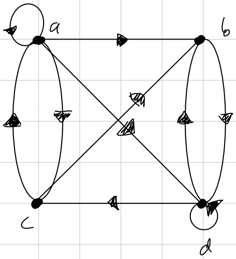
Vertex	adjacent
a	b, c, d
b	a, d
c	a, d
d	a, b, c

③



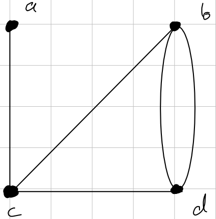
initial Vertex	Terminal Vertex
a	a, b, c, d
b	d
c	a, b
d	d, c, b

⑦ Represent the graph in exercise 3 as an adjacency matrix



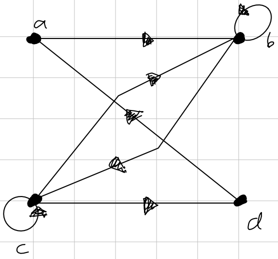
$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

⑬ Reprecent The graph as an adjacency matrix



$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \end{bmatrix}$$

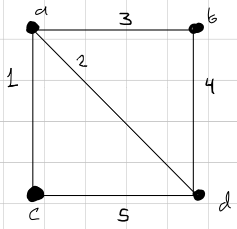
⑭ Find The adjacency matrix of the given directed multigraph



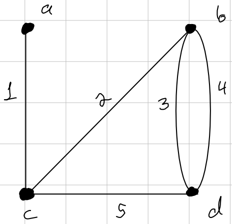
$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

→ Incident Matrix for Graps 1 and 13

#3



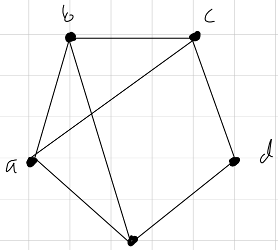
$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline a & 1 & 1 & 1 & 0 & 0 \\ b & 0 & 0 & 1 & 1 & 0 \\ c & 1 & 0 & 0 & 0 & 1 \\ d & 0 & 1 & 0 & 1 & 1 \end{array}$$



$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

• Section 10.5

① Determine whether the given path is an Euler circuit. Construct a circuit when one exists. If No Euler circuit exist determine whether it has an euler path and construct one if one exists.

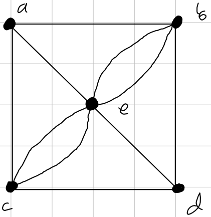


Vertex	degree
a	3
b	3
c	3
d	2
e	3

e

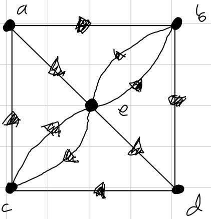
Not an Euler path or circuit

③



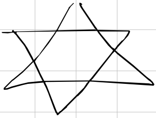
Vertex	Degree
a	3
b	4
c	4
d	3
e	6

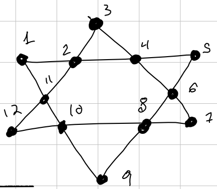
Euler Circuit



a → b → d → c → a → e → b → e → c → e → d

⑬ Determine whealther the picture can be drawn with a pensil in a continuos motion whiout lifting the pencil or retracing part of the picture

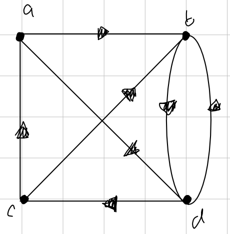




Vertex	Degree
1	2
2	4
3	2
4	4
5	2
6	4
7	2
8	4
9	2
10	4
11	4
12	2

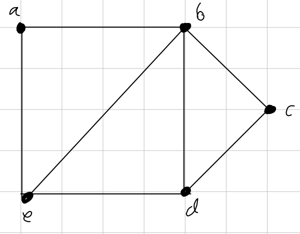
It cold

⑲ Determine whether the directed graph shown has an Euler circuit. Construct an euler circuit if one exist. If no Euler circuit exists, determine whether it has an Euler path, Construct one if it exists.



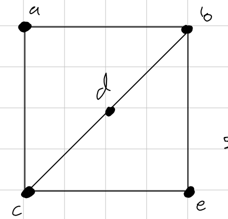
Neither

(31) Detemine if the graph has a hamilton circuit if it does find such circuit. If it doesnot give an argument why not.



a → b → c → d → e → a

(35)



No circuit
because it wont
satisfy theorems 3 and 4

(41) Does the graph in Exersise 35 have a hamilon path? If so, find such path if not, give an arguament why it doesnot exist?

I don't know.

- Section 10.6

(3)

See figure in book

$a \to c \to d \to e \to g \to z = 16$

$b \Rightarrow f$

$b \to f = 10$

(11) a chicago

(13) a chicago

(25) a - c - b - d - a and same but in reverse.