Calculus III BPR

- 14 Vector-Valued Functions

14 - Vector-Valued Functions

- Useful for objects in motion and arch length etc.
  - Same calculus things
- 14.1 Vector valued functions

14.1 Vector-Valued functions

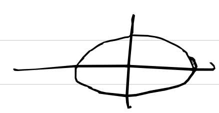
- A function of the form $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$
  - A set of three parametric equations that describe a curve in space
  - A vector valued function: there are 3 component variables (x, y, z), are components of $\vec{r}$, each component variable with respect to t.
- Circle in parametric
  - $x = a\cos t$, $y = a\sin t$ for $0 \le t \le 2\pi$
- Circle in Vector function
  $\vec{r} = \langle a\cos t, a\sin t \rangle$ for $0 \le t \le 2\pi$

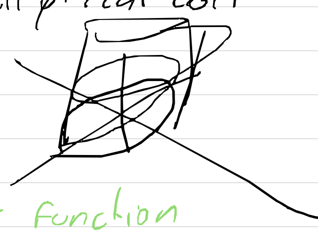
- Line

Given some points $\langle x, y, z \rangle$, $\langle x_0, y_0, z_0 \rangle$
we got $x = x_0 + at$ ... sub x by y or z
① say $\langle x - x_0, y - y_0, z - z_0 \rangle = \vec{PQ} = \vec{v}$ eg $(1, 1, \cancel{2})$
② Use P_0
$x = 2 + t$, $y = -1 + t$ ... $\vec{r}(t) = \langle x - 2t, \dots \rangle$
③ P_0 is parallell to $\vec{v}$

- Spiral

$\vec{r}(t) = 4\cos t\,\hat{i} + \sin t\,\hat{j} + \frac{1}{2\pi}\hat{k}$
① $z = 0$ to determine the projection of the plane
② since $0 \le t \le 2\pi$
z rises to 1
creating an eliptical coil





- Limit of a vector function

$\lim_{t \to a} \vec{r}(t)$

- how to use it

$\lim_{t \to a} \vec{r}(t) = \langle \lim_{t \to a} f(t), \lim_{t \to b} g(t), \dots \rangle$

For vectors in unit vector equation
Just evaluate.

B P R 15 Functions of Several Variables 18/3/25

- Partial derivatives, directional derivatives and gradient.
  - To find maximum and minimum of multivariable functions. Using tangent planes. Optimization problems.

- 15.1 Graphs and level curves
  - ASK! How do we define a "function" of several variables.

- Function of two variables
  $z = f(x, z)$ or implicitly $0 = f(x, y, z)$

- Deffinition Function, Domain, Range with two independent variables
  A function $z = f(x, y)$ assigns to each point (x, y) in a set D in $\mathbb{R}^2$ a unique real number z in a subset of $\mathbb{R}$. The D is the domain of f. The range is the set of real numbers z that are assumed as the points (x, y) vary of the domain.

17 Vector Calculus 6/5/25

- Chapter Preview
  - Covers everything seen previously endor calculus series. :')
- **17.1 Vector Fields**
- Vector field: A unique vector in each point in space

- Vector Fields in two dimentions
  - A short step to understanding vectors in $\mathbb{R}^3$
- Definition Vector fields in Two Dimensions
  Let f and g be defined on a region R of $\mathbb{R}^2$. A vector field in $\mathbb{R}^2$ is a function $\mathbf{F}$ that assigns each poin in R a vector $\langle f(x,y), g(x,y)\rangle$. The vector fields ar written as

$$\mathbf{F} = (x,y) = \langle f(x,y), g(x,y)\rangle$$
$$\mathbf{F} = (x,y) = f(x,y)\,\hat{i} + g(x,y)\,\hat{j}$$

A vector field $\mathbf{F} = \langle f, g\rangle$ is continuous or differentiable on a region R, respectively.

- E.g. $\mathbf{F}(x,y) = \langle 2x, 2y\rangle$

  $F(1,1) = \langle 2, 2\rangle$
  - This is a vector $\langle 2,2\rangle$ starting at $(1,1)$
- Flows

- Radial vector fields in $\mathbb{R}^2$ they point away from the origen parallel to the position vectors $r = \langle x, y\rangle$

$$\mathbf{F}(x,y) = \frac{\bar{r}}{|\bar{r}|^p} = \frac{\langle x,y\rangle}{|\bar{r}|^p} = \frac{\bar{r}}{|\bar{r}|}\frac{1}{|r|^{p-1}}$$

What exactly is p here?

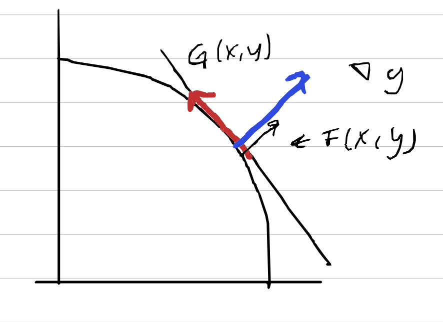
- Definitions Radial vector fields in $\mathbb{R}^2$
  Let $\bar{r} = \langle x, y\rangle$. A vector field or the form $F = f(x,y)\,\bar{r}$ where f is a scalar valued function, is a radial vector field. Of the specific interest are the radial vector fields

$$F(x,y) = \frac{\bar{r}}{|\bar{r}|^p} = \frac{\langle x,y\rangle}{|\bar{r}|^p}$$

where p is a real humber. At every point except the origin, the vectors of the field are directed outwards with a magnitude of $|F| = \frac{1}{|\bar{r}|^{p-1}}$

- E.g. Let C be the circle $x^2+y^2=a^2$ where $a>0$
  a) show that at each point C, the radial vector field $F(x,y) = \frac{\bar{r}}{|\bar{r}|} = \frac{\langle x,y\rangle}{\sqrt{x^2+y^2}}$ is orthogonal to the line tangent at C.

  The gradiant is ortogonal to the tangent at C. $\nabla g(x,y) = \langle 2x, 2y\rangle$



Ask!

How?

- **Vector Fields in Three Dimentions**
- Definition
  Let f, g, and h be defined on a region D of $\mathbb{R}^3$. A vector field in $\mathbb{R}^3$ is a function $\bar{F}$ that assingns to each point D a vector $\langle f(x,y,z), g(x,y,z), z(x,y,z)\rangle$.
  The vector field is written as.

$$\bar{F}(x,y,z) = \langle f(x,y,z), g(x,y,z), h(x,y,z)\rangle \text{ or}$$
$$\bar{F}(x,y,z) = f(x,y,z)\,\hat{i} + g(x,y,z)\,\hat{j} + h(x,y,z)\,\hat{k}$$

A vector field $\bar{F} = \langle f, g, h\rangle$ is continuos or differentiable on a region D of $\mathbb{R}^3$ if f, g, h are continuos or differentiable and D, respectively.
Of particular importance are radical vector fields

$$\bar{F}(x,y,z) = \frac{\bar{r}}{|\bar{r}|^p} = \frac{\langle x,y,z\rangle}{|\bar{r}|^p}$$

- **Gradient fields and Potential**
- A way to generate a vector field
- Start with a differentiable scalar valued function φ is called a gradient field and φ is called a potential function.
- Definition Gradiant fields and potential functions
  Let φ be differentiable on a region of $\mathbb{R}^2$ or $\mathbb{R}^3$. The vector field $\bar{F} = \nabla\varphi$ is a gradient field and the function φ is a potential function for $\bar{F}$

- So just find the gradiant is ta derivate like before?

- **Exponential curves and surfaces**
- equipotential curves (curves on which the potential function is constant
  Not sure!

- 17.6 Surface integrals