Section Summary Chapter 13 27-2-25

13.3 Cross Products

- Dot Product

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

where θ is the angle btween the vectors
- Angle between two vectors

$$\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \cos\theta$$

- Calculating a dot product

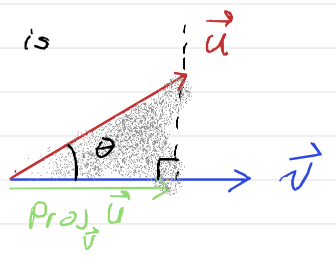
$\vec{u} = \langle a_1, a_2, \ldots, a_n\rangle$
$\vec{v} = \langle b_1, b_2, \ldots, b_n\rangle$

$$\vec{u}\cdot\vec{v} = a_1b_1 + a_2b_2 + \cdots + a_nb_n$$

- Orthogonal Projection

The scalar projection of $\vec{u}$ onto $\vec{v}$ is the magnitude of $proj_v\vec{u}$.

$|\vec{u}|\cos\theta = scal_v\vec{u}$



- Since θ is hard to find
It is used a unit vector in the direction of $\vec{v}$
and that is $\frac{\vec{v}}{|\vec{v}|}$

- Finding The projection of $\vec{u}$ onto $\vec{v}$

The projection of $\vec{u}$ on to $\vec{v}$ is given by

$$Proj_v\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v}$$

And the scalar is

$$scal_v\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}$$

$$Proj_{\vec{v}}\vec{u} = scal_v\vec{u}\cdot\frac{|\vec{v}|}{|\vec{v}|}$$

13.4 Cross Products

- Area of the paralelogram of two vectors
1 - Cross product $\vec{u}\times\vec{v}$
2 - Magnitude

$$A = |\vec{u}\times\vec{v}|$$

- Variation: Area of the triangle Given 3 points
1 → choose a pivot point
2 → Create 2 vectos
2.1 - $\vec{AB} = \langle b_1 - a_1, b_2 - a_2, b_3 - a_3\rangle$
$\vec{AC} = \langle \ldots \rangle$
3 → Apply the formul

$$\frac{|\vec{AC}\times\vec{AB}|}{2} = A_T$$

- Colinear

Poits are said to be colinear if they lie on the same line

- Verify if points are colinear
1 - Choose a pivot point
2 - Craft vectors using pivot
3 - Do a cross product of the vectors
- If the cross product is zero
The given points are colinear
- else
Not colinear.

13.5 Lines and plains in space

- Creating vector line equations that pass trough a particular point and are perpendicular to specific vectors and other given conditions.
- Pains. Vector plainequations
- these equations are parametric

- The line perpendicular to $\vec{a}$ and $\vec{b}$ trog P
1 - Find the vector perpendicular to $\vec{a}$ and $\vec{b}$
1.1 cross product, order wont matter.
2 - write the equation
$P + t(\vec{a}\times\vec{b})$
→ $\langle P_1, P_2, P_3\rangle + t\langle a_1, a_2, a_3\rangle$
→ The parametric equations are
$x = P_1 + a_1t,\ y = P_2 + a_2t,\ z = P_3 + a_3t$
- Note: The parameter of t is $\mathbb{R}$

- Parametric equation of the line going throgh P_1, P_2.
1 - Find $\vec{v} = \vec{P_1P_2}$
2 - write the equation
1.1 - compact
$\ell = P_1 + t\vec{v}$
2.2 individual
$x = p_1 + v_1t,\ y = p_2 + v_2t,\ z = p_3 + v_3t$
- Note: $t \in \mathbb{R}$

Variation 1: The line Joining the points P_1 and P_2
1 - Same steps, The only chaning is the parameter, Now going from 0, to 1
$0 \le t \le 1$

- Equation of a plane

a) → Given a point and a vector.
1 - Point given is $P(x_0, y_0, z_0)$
and vecto is $\vec{v} = \langle a, b, c\rangle$
2 - Replace the values in the equation
$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
- Note, dont forget the zero.

b) → Variation, Given 3 points.
1 - Create 2 vectors
$\vec{PQ}$, $\vec{PR}$
2 - $\vec{PQ}\times\vec{PR}$ That creates the neutral vecto
3 - back to a, choose P as your sh**!

- Equation of a plane parallel to two vectors
1 - Given $\vec{v}$ and $\vec{u}$ perform
$\vec{v}\times\vec{u} = \vec{c}$ → Order does not matter
where $\vec{c}$ is
2 - Evaluate $\vec{c}$ using P, replace the $\hat{i}, \hat{j}, \hat{k}$ with the members of P, d
3 - Rewrite $\vec{c}$, $\hat{i} = x$, $\hat{j} = y$, $K = z$
4 - Write the equation
$ax + by + cz = d$

Section Summary Chapter 14
— Vector Valued functions.

- Function of the line passing through P and Q
  1 – $P\langle x_1, x_2, x_3, \dots, x_n\rangle$
  $Q\langle y_1, y_2, y_3, \dots, y_n\rangle$

  2 → $r(t) = \langle Q - P\rangle$ where $t \in \mathbb{R}$, $(-\infty, \infty)$

  Ask! → Does the order matter?

  → My theory. No, it would just change the direction of the line.

- Distance of $\vec{r}(t)$ to a point P.
  1 – re write the components as

  $$(a - p_1)^2 + (b - p_2)^2 + \dots + (n - p_n) = D$$

  2 – Derivate that expression.
  3 – Set to 0, solve for t.
  3.1. The t is the length from
  4 – Length is when you plug into the original function

- Arch length Formula
  $$A = \int_a^b |r'(t)|\, dt$$

- Unit vector $\vec{T}(t)$
  1 – Derivate the $r'(t)$
  2 – Derivate by the magnitude of $|r'(t)|$
  3 – $\boxed{\dfrac{r'(t)}{|r'(t)|}}$

- Unit Normal vector $\vec{N}(t)$
  $$\boxed{\frac{T'(t)}{|T'(t)|}}$$

  – Where $T(t)$ is the unit normal vector.

- Derivatives of dot product and cross product
  $$\frac{d}{dt}[\vec{u}\cdot\vec{v}] = \vec{u}'\cdot\vec{v} + \vec{u}\cdot\vec{v}'$$
  $$\frac{d}{dt}[\vec{u}\times\vec{v}] = \vec{u}'\times\vec{v} + \vec{u}\times\vec{v}'$$

- Compute the derivative of $[\vec{u}\times\vec{v}]'$ or $[\vec{u}\cdot\vec{v}]'$
  – Just plug in values.

- 14.4
- Changing a parameter
  → Kind of like the inverse for a parametric equation

- Reparametrizing with curve length
  1 – Find $|r'(t)|$
  2 – integrate $\int^t |r'(t)|\, dt = R(t)$
  3 – solve $R(t) = S$
  4 → Replace all t's in the $\vec{r}(t)$ with s, to obtain $\vec{r}(s)$
  5 → Change the parameters
  $a_0 < t < b_0 \Longleftrightarrow a_1 < s < b_1$

- 14.5
  Curvature: How curvy a given function is, expressed by $\frac{1}{r}$ where r is the radius of the largest circle that can be fitted under/over the particular point of the curve.

- Curvature of a graph
  $$\kappa = \boxed{\frac{|T'(t)|}{|r'(t)|} = \frac{1}{R(t)}}$$
  1 – Find $r'(t)$, $T(t)$, $T'(t)$, $|r'(t)|$, $|T'(t)|$
  2 – Plug in.

  – Alternative formula for κ
  $$\kappa = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|^3}$$
  where $\vec{v} = \vec{r}'(t)$
  and $\vec{a} = \vec{r}''(t) = \vec{v}'$

- Unit Binomial vector $\vec{B}$
  $$\vec{T}\times\vec{N} = \vec{B}$$

- Torsion τ
  $$\tau = \frac{d\vec{B}}{ds}\cdot\vec{N} \qquad \text{where} \qquad \frac{\frac{dB}{dt}}{\frac{ds}{dt}} = \frac{d\vec{B}}{ds}$$
  and $\dfrac{ds}{dt} = |r'(t)|$

Section Summary
15 Functions of Several Variables

- 15.1 Graphs and level curves
- Implicit or Explicit form
  Implicit form: $F(x,y) = 0$
  Explicit form $y = f(x)$
  Explicit $z = f(x,y)$ Implicit $F(x,y,z) = 0$

- 15.2 Limits and continuity
  Same properties as one variable limits.

- Solving Multivariable limits
  1- Plug in variables
  1.1 if that fails try some algebraic manipulation like conjugation.
  2- Investigate the limit
  2.1 - write a function of x in terms of y or write a function of y in terms of x and replace the y's and x's respectively. when evaluating the single variable limit it should match the other limit. If not the limit does not exists

- Continuity:
  1- f is defined at (a,b)
  2- $\lim_{(x,y)\to(a,b)} f(x,y)$ exists
  3- $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

- 15.3 Partial Derivatives
- Notation

$$\frac{\partial f}{\partial x}$$

- Doing Partial Derivatives
  1- $\frac{\partial f}{\partial a}$ treat all other variables like constants and derivate like normal
  2- Second Derivative
  - The order does not matter

- 15.4 The chain Rule
- Doing the Chain Rule

$$\frac{\partial z}{\partial t} = \frac{dz}{\partial x}\frac{dx}{\partial t} + \frac{dz}{\partial y}\frac{\partial y}{\partial y}$$

1 - a function in terms of x and y will be given, then find its partial derivatives with respect to x and y.

2 - separate functions $x = f(t)$ and $y f(t)$ will be given, derivate.
3- Multiply like the formula says.
4- If more than 2 variables add one more "block"

- 15.5 Directional Derivatives and the Gradient
  At a point there are many rates of change to go from. so picking a direction and a point makes it less abstract.

- Directional Derivative
  1 Must have a point, and a unit vector and a function use the formula

$$D_{\vec{u}} f(a,b) = \langle f_x(a,b), f_y(a,b)\rangle \cdot \langle u_1, u_2\rangle$$

2 Calculate f_x, f_y and evaluate at P
3 Dot the two vectors together

- Gradient Vector
- Find the gradient
  $\nabla f(x,y) = \langle f_x(x,y), f_y(x,y)\rangle = f_x(x,y)\,\hat{i} + f_y(x,y)\,\hat{j}$

- Interpretations of the gradient
  Let f be differentiable at (a,b) with $\nabla f(a,b) \neq 0$
  1- f has a maximum rate of increase at (a,b) in the direction of the gradient $\nabla f(a,b)$. The rate of change of this direction is $|\nabla f(a,b)|$
  2- f has a maximum rate of decrease at (a,b) in the direction of $-f(a,b)$. The rate of change at this direction is $-|\nabla f(a,b)|$
  3- The directional derivative is zero in any direction orthogonal to $\nabla f(a,b)$

- The gradient level curves
  ASK!

- 15.6 Tangent Planes and Linear Approximation
- Finding tangent planes of functions
  1 - find the gradient ∇F at P
  2 - Dot it with a vector $\langle x-a, y-b, z-c\rangle$ to obtain

$$f_x(a,b,c)(x-a) + F_y(a,b,c)(y-b) + F_z(a,b,c)(z-c) = 0$$

- On linear approximation
  - Is just the plane tangent at a point P.

- 15.7 Maximum and Minimum problems

- Unbounded Maximum and Minimum
  1 Find the gradiant ∇f
  2 Set each partial derivative to zero
  2.1 Do not Divide anything
  3 Solve for the variables
  3.1 - Plug in to other equations to get points

  4 Second Derivative test
  4.1 find every second Derivative and evalute using the formula

$$D(x,y) = f_{xx}(x,y) f_{yy}(x,y) - (f_{xy}(x,y))^2$$

- If $D(a,b) > 0$ and $f_{xx}(a,b) < 0$, then f has a local maximum value at (a,b).
- If $D(a,b) > 0$ and $f_{xx} > 0$, then f has a local minimum value at (a,b).
- If $D(a,b) < 0$, then f has a saddle point at (a,b)
- If $D(a,b) = 0$ then inconcusive.

- Absolute Maximum and absolute Minimum.
  use Lagrange Multipliers

- 15.8 Lagrange Multipliers
  For finding Maxima and minima under a volume or region

- Finding Max / Min using Lagrange multipliers
  1 - Find $\nabla f(x,y,z)$
  2 - Find $\nabla g(x,y,z)$
  3 - Solve

$$f_y = \lambda g_y$$
$$f_x = \lambda g_x$$
$$f_z = \lambda g_z$$
$$g(x,y,z) = 0$$

3.1 Do not Divide
3.2 when taking roots account for both sides
4 Find Points by evaluating the other equations

Section Summary Chapter 16 20-9-25

16 Multiple Integration

- **16.1 Double integrals over Rectangular regions**

Volume of a solid is estimated with rectangular boxes

- Region $R = \{(x,y);\ a \le x \le b,\ c \le y \le d\}$
  those are the bounds of the integral

- Reimann sum compared to integral

$$\iint_R f(x,y)\,dA = \lim_{\Delta \to 0} \sum_{k=1}^{n} f(x_k^*, y_k^*)\,\Delta A_k$$

- **Iterated Integrals**
  - Goes in layers like an onion.
- **Fubini Integral**

$$\iint_R f(x,y)\,dA = \int_a^b\int_c^d f(x,y)\,dx\,dy = \int_c^d\int_a^b f(x,y)\,dy\,dx$$

- The order can be changed.

- **Average Value of a function over a plane region.**

$$\bar{f} = \frac{1}{\text{Area of } R}\iint_R f(x,y)\,dA$$

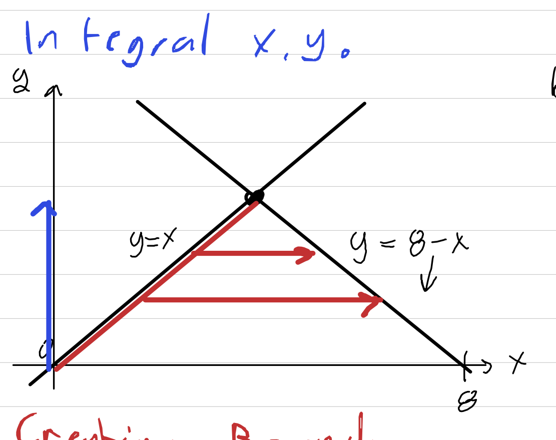
- **16.2 Double integrals over general regions**

Instead of integrating over easy rectangles, it's hard now

- **Double integral over nonrectangular regions**

$$\iint_R f(x,y)\,dA = \int_a^b\int_{g(y)}^{h(y)} f(x,y)\,dy\,dx = \int_d^c\int_{g(x)}^{h(x)} f(x,y)\,dx\,dy$$

- **Integral x, y.**



blue integration with dx
Red integration with dy

- **Creating Bounds**
  1- Graph the region bounded
  2- Try to find un interrupted lines, where the bottom function and top function do not disconnect.
  3- First bound.
  The inner integral, if integrating dy moves from left to right, if integrating dx moves from bottom to top
  3.1 dy
  → Re write the most left and right functions in terms of x
  → if dx
  same but use y
  4- Outer bound.
  - From the lowest value of y or x in the region to the largest
  5- It might have to be split

- Integration process is simple

- **16.3 Double integrals in Polar Coordinates**
  - From rectangular to polar
    $r\cos\theta = x$, $r\sin\theta = y$

    $dA = r\,dr\,d\theta$

- **Integral Polar**

$$\iint_R f(x,y) = \int_\alpha^\beta\int_a^b f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

- Polar coordinates have a strict order

- **Polar integrals**
  1- Going from rectangular to polar
  $x = r\cos\theta$
  $y = r\sin\theta$

  2- Integration Regions
  2.1 graph
  2.2 Find the smallest radius
  - May be a function of r in terms of θ
  2.3 - Smallest angle to largest angle
  No functions

  3- Set up the integral

$$\iint_D f(x,y)\,dA \to \int_{\theta_1}^{\theta_2}\int_{r_1}^{r_2} f(r,\theta)\,r\,dr\,d\theta$$

  in that particular order

- Note, enclosed regions go

$$f(x) \geq g(x) \qquad \iint_A g(x) - f(x)\,dA$$

- **16.4 triple integrals**

- **Triple integral process**
  1 There are multiple ways to evaluate
  - Pic a coordinate to integrate
  dz
  Make sure it gives the bounds in terms of another variable
  2 - Flatten the image
  - Do double integration of that shape.

- **16.5 Triple integrals of spherical and Cylindrical coordinates**
- **Transforming into cylindrical coordinates**

Rectangular to cylindrical | Cylindrical → Rectangular

Rectangular to cylindrical	Cylindrical → Rectangular
$r^2 = x^2 + y^2$	$x = r\cos\theta$
$\tan\theta = y/x$	$y = r\sin\theta$
$z = z$	$z = z$

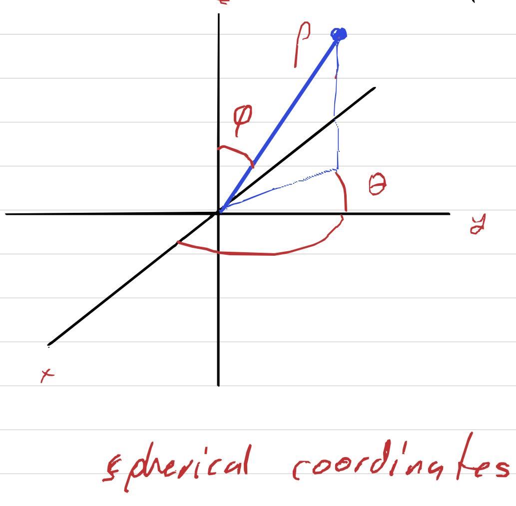
- **Integrating cylindrical coordinates**

$$\iiint_D f(x,y,z)\,dA = \int_\alpha^\beta\int\int_{H(r\cos\theta,\, r\sin\theta)}^{G(r\cos\theta,\, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\,dz\,r\,dr\,d\theta$$

- **Transform spherical coordinates**

rectangular → spherical
$\rho = x^2 + y^2 + z^2$

spherical → rectangular
$x = \rho\sin\varphi\cos\theta$
$y = \rho\sin\varphi\sin\theta$
$z = \rho\cos\varphi$



- **Integral spherical coordinates**

$$\iiint_D f(x,y,z)\,dV =$$

$$\int_\alpha^\beta\int_a^b\int_{g(\varphi,\theta)}^{h(\varphi,\theta)} f(\rho\cos\theta\sin\varphi, \rho\sin\varphi\sin\theta, \rho\cos\varphi)\,\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

Section Summary 17

Vector calculus — 28 May

- **17.1 Vector Fields**
  - Not much material

- **17.2 Line Integrals**

Evaluating the line integral $\int_C f\,ds$

1- Find the parametric description of C in the form
$\bar r(t) = \langle x(t), y(t)\rangle$ for $a \le t \le l$

2- compute $|\bar r'(t)| = \sqrt{x'(t) + y'(t)}$

3- Make substitutions in the integrand

$$\int_C f\,ds = \int_a^b f(x(t), y(t))\,|\bar r'(t)|\,dt$$

Extra

The region $\bar r, \langle a, b\rangle$ goes in f

$f(\bar r(a,b))$

- **Scalar line integrals**

1 Create a parametric description of a line connecting two points P, Q

$P(a,b,c)$, $Q(x,y,z)$

$\bar r(t) = \langle a,b,c\rangle + t\langle x,y,z\rangle \quad 0 \le t \le 1$

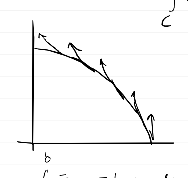
$= \langle a+tx, b+ty, c+tz\rangle$

Do previous system

- **Line integrals of vector fields**

Let $\bar F$ be a vector field that is continuos on a region containing a smooth oriented curve C parametrized by arch leinght. Let $\bar T$ be the unit tangent vector at each point of C consistent with orientation. The line integral of $\bar F$ over c is

$$\int_C \bar F \cdot T$$



Recall the unit tangent vector is $\dfrac{r'(t)}{|r'(t)|}$

$$\int_C \bar F \cdot \bar T\,ds = \int_a^b \bar F \cdot \frac{\bar r'(t)}{|\bar r'(t)|}\,|\bar r'(t)|\,dt$$

$$\int_a^b \bar F \cdot \bar r'(t)\,dt$$

- **Types of line integrals of vector fields**

$\bar F = \langle f, g, h\rangle$ C has a parametrization

$\bar r(t) = \langle x(t), y(t), z(t)\rangle$ for $a \le t \le b$

$$\int_a^b \bar F \cdot \bar r'(t)\,dt = \int_a^b (f(t)x'(t) + g(t)y'(t) + h(t)z'(t))\,dt$$

$$= \int_C f\,dx + g\,dy + h\,dz$$

$$= \int_C \bar F \cdot d\bar r$$

- **Evaluating line integrals over a vector field**

1- Parametrize the line
2- Find $r'(t)$
3- Fin $F(r(t))$
4-

$$\int f\,dx + g\,dy + h\,dz$$

- **Circulation and Flux of a vector field**

The circulation is a measure of how much the vectorfield pushes in the direction of C.

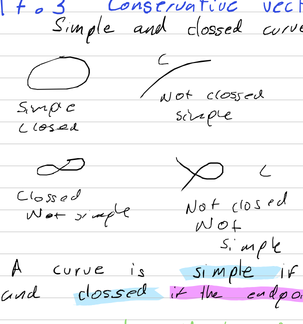
The flux is the cross wind positive (left) negative right)

- **Flux integral**

$$\int_C \bar F \cdot \bar n\,ds = \int_C f\,dy - g\,dx$$

- **17.3 Conservative vector fields**

Simple and clossed curves



A curve is simple if it never intersects and clossed if the endpoints match.

- **Conservative Vector Field**

A vector field is conservative on a region if there exist a scalar function such that

$$\bar F = \nabla \varphi \quad \text{on that region.}$$

- **Test for conservative vector fields**

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}, \quad \frac{\partial f}{\partial z} = \frac{\partial h}{\partial x} \quad \frac{\partial g}{\partial z} = \frac{\partial h}{\partial y}$$

see how they are opposites
$f \Leftrightarrow x \quad g \Leftrightarrow y \quad h \Leftrightarrow z$

$\dfrac{\partial f}{\partial y}$ $\dfrac{\partial g}{\partial x}$

- **Fundamental theorem of line integrals**

$$\int_C \bar F \cdot T\,ds = \int_C \bar F \cdot d\bar r = \varphi(B) - \varphi(A)$$

- **Finding the potential function**

1- Integrate f with respect to x
$\varphi + C(y,z)$

2- Derivate with respect to y
$\varphi_y = g$
solve for $C(y,z)$
2.1 integrate $C(y,z)$ to y

3. Replace $C(y,z)$
Derivate with respect to z
3.1 set equal $\varphi_z = h$
solve, integrate

4- φ

- **Evaluating integrals using TFTLI**

1- obtain the potential function φ
2- evquate $\bar r$ by the endpoints
3- Plug in into φ.

- **17.4 Green's theorem**
- **Green's theorem Circulation form**

For a simple clossed curve, conservative

$$\oint_C \bar F \cdot d\bar r = \oint_C f\,dx + g\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA$$

- **Two Dimentional curl**

For $F = \langle f, g\rangle \Leftrightarrow \dfrac{\partial g}{\partial x} - \dfrac{\partial f}{\partial y}$

- **Green's theorem Flux form**

$$\oint_C \bar F \cdot \bar n\,ds = \oint_C f\,dy - g\,dx = \iint_R \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dA$$

- **See the difference of Flux and Circulation.**

- **Evaluating circle integrals**

1- What are you dealing with
Types: Flux and Circulation

1.1 - Circulation

$$\oint_C \bar F \cdot d\bar r = \oint_C f\,dx + g\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA$$

1.2 - Flux

$$\oint_C \bar F \cdot \bar n\,ds = \oint_C f\,dy - g\,dx = \iint_R \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dA$$

2- Finding the functions
only hard if given in $f\,dy$, $g\,dx$
ex.

$$\underbrace{(10x + e^{y\ln y})}_{f}\,dy - \underbrace{(7y^7 + e^{x\ln x})}_{g}\,dx$$

3- Bounds if given as points
- Construct a diagram
- Do normal techniques to come up with the bounds

4- Possibly switch to polar etc.

- **17.5 Divergence and Curl**
- **Divergence of a vector field**

$$\text{div}\,\bar F = \nabla \cdot \bar F = \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial h}{\partial z}$$

- **Curl of a vector field**

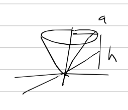
$$\nabla \times \bar F = \text{curl}\,\bar F$$

$$\left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}\right)\hat i + \left(\frac{\partial f}{\partial z} - \frac{\partial h}{\partial x}\right)\hat j + \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)\hat k$$

- **17.6 Surface integrals**
- **Parametric Descriptions of curves**

Plain
set $x = u$, $y = v$, $z(u,v)$

$\bar r(u,v) = \langle u, v, z(u,v)\rangle$



Cylinders

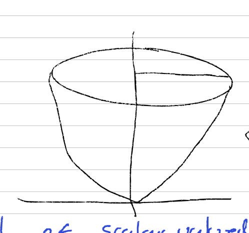
$\theta = u \quad v = z$

$\left\langle \dfrac{av}{h}\cos u, \dfrac{av}{h}\sin u, v\right\rangle$

Just like in cylindrical

Spheres

$\langle a\sin u\cos v, a\sin u\sin v, a\cos u\rangle$



$\langle \sqrt v\cos u, \sqrt v\sin u, v\rangle$

- **Integral of scalar valued functions**
- **Solving steps**

1- Parametrize the region
2- evaluate the function with the paramelers
3- Find the bounds
4- take the r and call it t
4.1

$$t_u = \frac{\partial \bar r}{\partial t} = \left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial u}\right\rangle$$

$$t_v = \frac{\partial \bar r}{\partial t} = \left\langle \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}\right\rangle$$

4.2

$$|t_u \times t_v|$$

5- Do the integral

$$\iint_R f(x(u,v), y(u,v), z(u,v))\,|t_u \times t_v|\,dA$$

- **Integrals on explicitly defined surfaces.**

an explicit description is $z = g(x,y)$

Solving steps

1- Verify is in explicit form
- basically the whole thing could be expressed without z

2- Find the t's
2.1 z_x
2.2 z_y
2.3 $\sqrt{z_x^2 + z_y^2 + 1}$

3- Rewrite the interior of the integral only using x and y.

4- Find the bounds
set the equation of S, the region expressed as $z = g(x,y)$
4.1 set $z = 0$
4.2 graph
4.3 create bands

5- Put everything together
5.1 like this

$$\iint f(x, y, g(x,y))\,\sqrt{z_x^2 + z_y^2 + 1}\;dA$$

- **Flux integral**
  - The surface integral of a vector field

$$\iint_S \bar F \cdot \bar n\,dS = \iint_R \bar F \cdot (\bar t_u \times \bar t_v)\,dA$$

where $\bar t_u = \dfrac{\partial \bar r}{\partial u} = \left\langle \dfrac{\partial x}{\partial u}, \dfrac{\partial y}{\partial u}, \dfrac{\partial z}{\partial u}\right\rangle$

$\bar t_v = \dfrac{\partial \bar r}{\partial v} = \left\langle \dfrac{\partial x}{\partial v}, \dfrac{\partial y}{\partial v}, \dfrac{\partial z}{\partial v}\right\rangle$

If S is defined explicitley
$z = s(x,y)$ for (x,y) in a region R
then

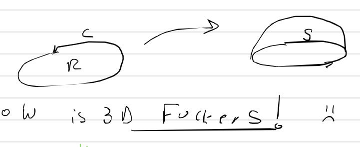
$$\iint_S \bar F \cdot \bar n\,ds = \iint_R (-f z_x - g z_y + h)\,dA$$

- The z function is the surface.

- **17.7 Stroke's theorem**

The 3 dimention form of version of the circulation form of green's theorem

$$\underbrace{\oint_C \bar F \cdot d\bar r}_{\text{circulation}} = \underbrace{\iint_R (g_x - f_y)\,dA}_{\text{curl or rotation}}$$



Now is 3D Fuckers! ☹

- **Stroke's theorem**

$$\oint_C \bar F\,d\bar r = \iint_S (\nabla \times \bar F) \cdot \bar n\,dS.$$

- **Solving Problem with it**

1- 2 main ways
2- If the surface can be parametrized then
2.1 $\oint_C F \cdot dr = \int_a^b F(\bar r(t)) \cdot r'(t)\,dt$

3- Cant be parametrized
Do

$$\oint_C \bar F \cdot d\bar r \Rightarrow \iint_S \underbrace{(\nabla \times \bar F)}_{G} \cdot \bar n\,ds$$

$$\Rightarrow \iint_S G \cdot n\,ds$$

if the plaine is given in explicit form, thank god do

$$\iint_R (-f z_x - g z_y + h)\,dA$$

if not

$$\iint F \cdot (t_u \times t_v)\,dA$$

Parametrize the surface

- **17.8 Divergence Theorem**

Flux form of green's theorem → Divergence theorem

- **Divergence theorem**

$$\iint_S \bar F \cdot \bar n\,dS = \iiint_D \nabla \cdot \bar F\,dV$$