Practice Problems for Exam 1

Note: This list of practice problems is much longer than the exam will be. I just wanted to give you plenty to practice on.

1. Perform the following operations on the indicated vectors. If an operation is not possible, state why.

$$\mathbf{u} = \langle -2, -3, 1 \rangle \qquad \mathbf{v} = \langle 0, 2, 3 \rangle \qquad \mathbf{w} = \langle -3, 2, 4 \rangle$$

a) $2\mathbf{u} + 3\mathbf{v}$
b) $-\mathbf{u} + \mathbf{v} + 2\mathbf{w}$
c) $\mathbf{u} - 2\mathbf{v} - 4\mathbf{w}$
d) $\mathbf{u} \cdot \mathbf{v}$
e)
g)
h) $\mathbf{u} \times \mathbf{v}$
i) $\mathbf{u} \times \mathbf{w}$
j) $\mathbf{w} \times \mathbf{v}$
k) $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$
l) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$
m) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$
o) Find the area of the parallelogram formed by $\mathbf{v}$ and $\mathbf{w}$.

2. Find the vector form parametric equations of the following lines.

a) The line through the point $(1, 2, -3)$ and parallel to $2\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$.
b) The line passing through the points $(2, 1, 2)$ and $(2, -3, 5)$.
c) The line passing through the points $(3, -5, 1)$ and $(0, 2, 7)$.

→ set equal, solve system of linear equations evaluate

$\vec{PQ}$ $\vec{PR}$ → $\vec{PQ} \times \vec{PR}$ = $\vec{n}$ same before

a) The plane containing the point $(7, 2, 4)$ with normal vector $\vec{n} = \langle -1, 2, 5 \rangle$
b) The plane containing the points $P(2, 0, -1)$, $Q(3, -4, 4)$, and $R(-1, 5, 2)$.

$\vec{r}(t) = \langle 4\cos t, 4\sin t, 2t \rangle$ on $[0, 6\pi]$

$\vec{r}(t) = \left\langle \frac{t^3}{3}, t^2, 2t \right\rangle$ on $[0, 2]$

a) $\vec{r}(t) = \langle t + 1, \cos t, \sin t \rangle$
b) $\vec{r}(t) = \langle t, t^2 \rangle$

Practice Problems for Exam 1 - Answer Key

These are not fully worked-out solutions, but should allow you to check your work. These would, generally, not be enough to get full credit on a question.

1. a) $\langle -4, 0, 11\rangle$
   b) $\langle -4, 9, 10\rangle$
   c) $\langle 10, -15, -21\rangle$
   d) -3
   e) Can't do
   f) $\left\langle 0, -\frac{6}{13}, -\frac{9}{13}\right\rangle$
   g) $\left\langle \frac{3}{7}, \frac{9}{14}, -\frac{3}{14}\right\rangle$
   h) $\langle -11, 6, -4\rangle$
   i) $\langle -14, 5, -13\rangle$
   j) $\langle -2, 9, -6\rangle$
   k) 29
   l) $\langle 32, 56, -4\rangle$
   m) $\langle -9, 14, 24\rangle$
   n) $\cos^{-1}\left(\frac{4}{\sqrt{14}\sqrt{29}}\right)$
   o) 11

2. a) The vector form is $\vec{r}(t) = \langle 1 + 2t, 2 - 4t, -3 + 3t\rangle$
   b) One vector form is $\vec{r}(t) = \langle 2, 1 - 4t, 2 + 3t\rangle$
   c) One vector form is $\vec{r}(t) = \langle 3 - 3t, 7t - 5, 6t + 1\rangle$

3. $(3, -7, 4)$

4. a) One possible equation is $-(x - 7) + 2(y - 2) + 5(x - 4) = 0$
   b) One possible equation is $-37(x - 2) - 18y - 7(z + 1) = 0$

5. Find the lengths of the following curves on the given interval.
   a) $12\sqrt{5}\pi$
   b) $\frac{20}{3}$

6. Find the unit tangent and normal vectors for the following curves.
   a) $T(t) = \left\langle \frac{1}{\sqrt{2}}, -\left(\frac{\sin t}{\sqrt{2}}\right), \frac{\cos t}{\sqrt{2}}\right\rangle$
      $N(t) = \langle 0, -\cos t, -\sin t\rangle$
   b) $T(t) = \left\langle \frac{1}{\sqrt{4t^2+1}}, \frac{2t}{\sqrt{4t^2+1}}\right\rangle$
      $N(t) = \left\langle \frac{-2t}{\sqrt{4t^2+1}}, \frac{1}{\sqrt{4t^2+1}}\right\rangle$

1. $\vec{u} = \langle -2, -3, 1 \rangle, \vec{v} = \langle 0, 2, 3 \rangle, w = \langle -3, 2, 4 \rangle$

(a) $2\vec{u} + 3\vec{v}$

$\langle -4, -6, 2 \rangle + \langle 0, 6, 9 \rangle =$

$= \langle -4, 0, 11 \rangle$

(b) $\vec{u} - 2\vec{v} - 4\vec{w}$

$\langle -2, -3, 1 \rangle$

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g - $Proj_{\vec{u}} \vec{v}$

$Proj_{\vec{a}} \vec{b} = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \vec{a}$

$Proj_{\vec{u}} \vec{v} = \frac{\langle 0, 2, 3 \rangle \cdot \langle -2, -3, 1 \rangle}{|-2, -3, 1|^2} \vec{u}$

$0 \quad -6 \quad 3 = -3$

$\frac{(0)(-2) + (2)(-3) + (3)(1)}{(-2)^2 + (-3)^2 + (1)^2} \langle 0, 2, 3 \rangle = -\frac{3}{14} \vec{u}$

$4 \quad 9 \quad 1 = 14$

f. $Proj_v u$ $\quad \frac{-3}{13} \vec{v}$

$\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \cos\theta$

$u \langle -2, -3, 1 \rangle \quad w \langle -3, 2, 4 \rangle$

$6 - 6 + 4 = 4$

$\cos^{-1}\left(\frac{4}{\sqrt{14} \cdot \sqrt{29}}\right)$

o $\vec{v}\ \vec{w}$

$\begin{Vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 3 \\ -3 & 2 & 4 \end{Vmatrix} = \underbrace{8-6}_{2\hat{i}} - 9\hat{j} + 6\hat{k}$

$\sqrt{9^2 + 6^2} = 81 + 36$

2 - a $P(1, 2, -3)$ $\quad 2\hat{i} - 4\hat{j} + 3\hat{k}$

$\vec{r}(t) = \langle 1 + 2t, 2 - 4t, 3t - 3 \rangle$

b) the line pass. $P(2, 1, 2)$ $Q(2, -3, 5)$

$\vec{PQ} = \langle 0, -4, 3 \rangle$

$\vec{r}(t) = \langle 2, 1 - 4t, 2 + 3t \rangle$

$t \in \mathbb{R}$ where $x = 2, y = 1 - 4t, z = 2 + 3t$

$\vec{PQ}$ $(3, -5, 1)$, $(0, 2, 7)$

$\vec{PQ} \langle -3, 7, 6 \rangle$

$\vec{r}(t) = \langle 3 - 3t, -5 + 7t, 1 + 6t \rangle$

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3. $\vec{r}(t) = \langle 6t, 2t - 8, 4t + 2 \rangle$
$\vec{s}(u) = \langle 7 - 2u, 3u - 13, 6 - u \rangle$

$\begin{cases} 6t = 7 - 2u \\ 2t - 8 = 3u - 13 \end{cases}$

$6t = 7 - 2u$
$t = \frac{7}{6} - \frac{1}{3}u$

$4(\frac{1}{2}) + 2 = 6 - 2$
$2 + 2 = 4$

$6x = 7 - 2y$
$x = \frac{7}{6} - \frac{1}{3}y$

$2x - 8 = 3y - 13$

$\frac{7}{3} - \frac{2}{3}y - 8 = 3y - 13$

$\frac{7}{3} - 8 + 13 = 3y + \frac{2}{3}y$

$\frac{7}{3} + 5 = \frac{11}{3}y$

$\frac{7 + 15}{3} = y$

$\frac{22}{3} = \frac{11}{3}y$

$\boxed{2 = y}$

$6x = 7 - 2(2)$
$6x = 7 - 4$
$6x = 3$
$\boxed{x = \frac{1}{2}}$

$(7 - 4, 6 - 13, 6 - 2)$

$(3, -7, 4)$

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4 (a) $\quad 0 = \langle -(x - 7) + 2(y - 2) + 5(z - 4)$

b) $P(2, 0, -1)$, $Q(3, -4, 4)$, $R(-1, 5, 2)$

$\vec{PQ}\ (1, -4, 5)$

$0 = (x + 1) - 4(y - 5) + 5(z - 2)$

$s = \int_a^b |r'(t)|\, dt$

$r(t) = \langle 4\cos t, 4\sin t, 2t \rangle \ [0, 6\pi]$

$\langle -4\sin t, 4\cos t, 2 \rangle$

$\int_0^{6\pi} \sqrt{(-4\sin t)^2 + (4\cos t)^2 + 4}\ dt$

$\int_0^{\pi/6} \sqrt{16\sin^2 t + 16\cos^2 t + 4}\ dt$

$\int_0^{\pi/6} \sqrt{16(\cos^2 + \sin^2) + 4}\ dt$

$\int \sqrt{16 + 4}\ dt$

$\int_0^{6\pi} \sqrt{16 + 4}\ dt = \sqrt{20}\int_0^{6\pi} dt = 6\pi\sqrt{20}$
$12\pi\sqrt{5}$

$\vec{r}(t) = \langle \frac{t^3}{3}, t^2, 2t \rangle \ [0, 2]$

$r'(t) = \langle t^2, 2t, 2 \rangle$

$\int_0^2 \sqrt{t^4 + 4t^2 + 4}\ dt$

$\int_0^2 \sqrt{(t^2 + 2)^2}\ dt$

$\int_0^2 t^2 + 2\ dt = \frac{1}{3}t^3 + 2t \Big|_0^2$

$\frac{8}{3} + 4 - 0 = \frac{20}{3}$

6 — $\vec{r}(t) = \langle t + 1, \cos t, \sin t \rangle$

7 → $\vec{r}(t) = \langle t, t^2 \rangle$

$r'(t) \langle 1, -\sin t, \cos t \rangle$

$\frac{r'(t)}{|r'(t)|} = \vec{u}(t) = \langle \frac{\sqrt{2}}{2}, \frac{-\sqrt{2}\sin t}{2}, \frac{\sqrt{2}\cos t}{2} \rangle$

$|r'(t)| = \sqrt{1 + \sin^2 t + \cos^2 t} = \sqrt{2}$

$r''(t) = \langle 0, -\cos t, -\sin t \rangle$

$T(t) = \frac{\langle 0, -\cos t, -\sin t \rangle}{1} = \langle 0, -\cos t, -\sin t$

$\vec{r}'(t) = \langle 1, 2t \rangle$

$\sqrt{1^2 + (2t)^2} = |r'(t)|$

$\vec{u}(t) = \langle \frac{\sqrt{1 + 2t^2}}{\ldots 2}, \frac{2t^2\sqrt{1 + 4t^2}}{\ldots t^2} \rangle$